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BENJAMIN PEIRCE

1809–1880

BIOGRAPHICAL SKETCH AND BIBLIOGRAPHY

BY

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REMINISCENCES

BY

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President A. Lawrence Lowell

Professor Emeritus W. E. Byerly
Harvard University

Chancellor Arnold B. Chace
Brown University

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FOREWORD

This monograph was originally published in the *American Mathematical Monthly,* January, 1925. The Mathematical Association of America issues it in separate form, with four new portraits and additional notes, so as to meet the needs of libraries and individuals throughout the world, interested in the outstanding figure in American Mathematics during the first three quarters of the nineteenth century.

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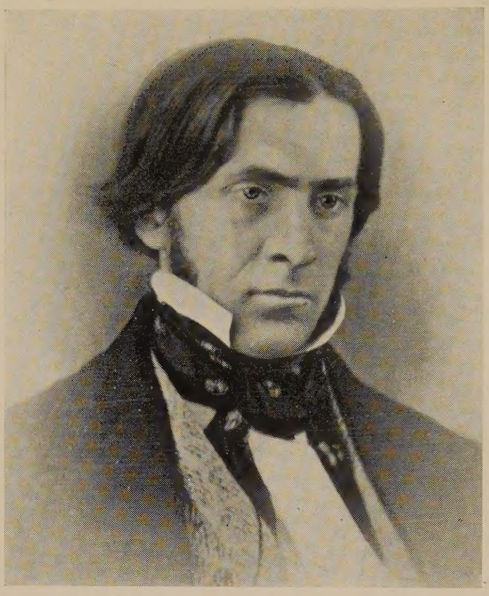
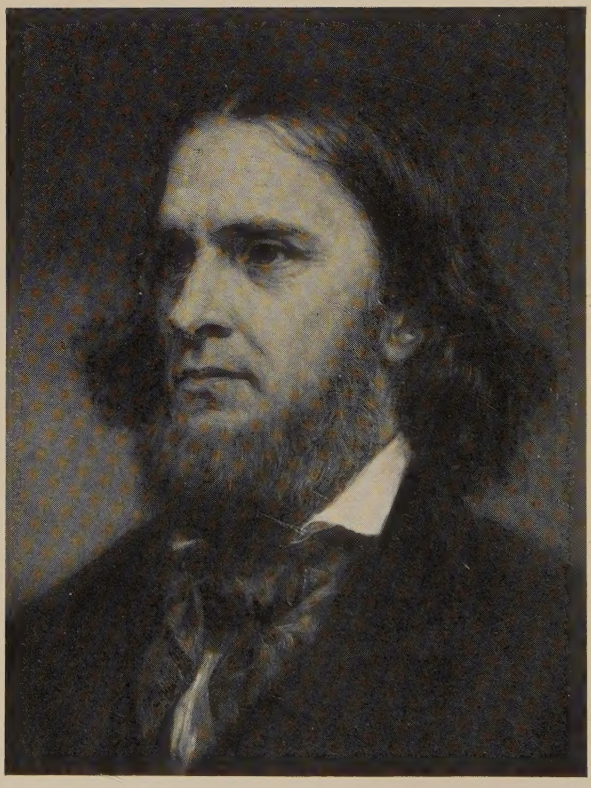


BENJAMIN PEIRCE, 1845 (?)

Earliest known portrait from a hitherto unpublished daguerreotype in the possession of his grandson, Benjamin P. Ellis.



BENJAMIN PEIRCE, 1857

From a hitherto unpublished painting by Daniel Huntington. Reproduced through the courtesy of George A. Plimpton of New York City, who has recently presented the portrait to Harvard University.

BENJAMIN PEIRCE

I. Reminiscences of Peirce.

By President Emeritus Charles W. Eliot,[1] Harvard University.

Benjamin Peirce graduated at Harvard College with the degree of A.B. in 1829. Two years later he was appointed Tutor and in 1833 University Professor of Mathematics and Natural Philosophy. This was an unendowed professorship; and its creation was one of President Quincy's enterprising adventures in the enlargement of Harvard's teaching staff. The President was doubtless

[1] The authors of the "Reminiscences" of Benjamin Peirce, presented herewith, were all his former students and each has done something notable in mathematics. President Emeritus Eliot was a student during 1849–53. He was a tutor of mathematics in Harvard College 1854–58, and assistant professor of mathematics 1858–61; he was also assistant professor of chemistry 1858–63. James Mills Peirce, son of Benjamin, and classmate of Eliot, was appointed tutor of mathematics at the same time. In his *Analytic Geometry*, published in 1857, Tutor Peirce acknowledged that "whatever merit the book may have is owing, in a great degree, to the assistance of Mr. C. W. Eliot." President Eliot has described these early years as follows (*Report of the Harvard Class of 1853*, Cambridge, 1913, p. 98): "Tutor Peirce chose the Freshman class, leaving me the Sophomore class in that year [1854–55]. After a year's experience, we applied some new recitation-room methods which made the mathematical instruction more effective. Finding the existing method of conducting oral examinations twice a year in the presence of visiting committees of the Board of Overseers very unsatisfactory as a test of the students' knowledge and capacity, we asked leave of the Faculty to conduct the mathematical examinations of the Freshmen and Sophomores in writing. After a good deal of hesitation the Faculty granted us leave to make the experiment; and these examinations were the first examinations in writing ever conducted for entire classes in Harvard College. The innovation was gradually adopted in other departments, and ultimately spread to the whole University.

"I tried to make the teaching of mathematics to the Freshmen and Sophomores as concrete as possible, and to illustrate its principles with practical applications. For example, while the class was studying trigonometry, I taught simple surveying to a group of volunteers, and with their help made a survey of the streets and open spaces of that part of Cambridge which lies within a mile and a half of University Hall. These volunteers made under my direction a careful map of what was then the College Yard, with every building, path, and tree delineated thereon—a map which is preserved in the college library."

President Lowell was a student under Peirce 1873–77, and his paper on "Surfaces of the second order as treated by quaternions," read before the American Academy of Arts and Sciences, was published in its *Proceedings* (vol. 13, 1878, pp. 222–250).

Professor Byerly was a student under Peirce 1867–71. He was assistant professor of mathematics at Cornell University 1873–76; assistant professor of mathematics at Harvard College 1876–81; professor 1881–1906; Perkins professor of mathematics 1906–1913. Since 1913 he has been Perkins professor emeritus. He was the first one (in 1873) to receive the degree of Doctor of Philosophy, in mathematics, at Harvard University; his thesis was entitled "The heat of the sun." He is the author of mathematical articles, pamphlets, and textbooks.

Chancellor Chace studied with Peirce in 1878–79, and his paper on "A certain class of cubic surfaces treated by quaternions," published in the *American Journal of Mathematics* (vol. 2, 1879, pp. 315–323), was a result. The Chancellor has recently prepared a translation, with commentary and notes, of the Rhind mathematical papyrus, which is about to be sent to the press.

R. C. Archibald.

supported in this adventure by Nathaniel Bowditch, who was then at the height of his influence as a Fellow of the Corporation (the President and Fellows of Harvard College). As soon as the endowed Perkins Professorship of Astronomy and Mathematics was established (1842), Benjamin Peirce was transferred to that chair, which he held till his death in 1880.

Benjamin Peirce was never a professor of Mathematics only. In the title of the University professorship he held, the broad subject of Natural Philosophy appeared, and in the title of the Perkins professorship Astronomy was the first subject named. These titles represented the real breadth of Benjamin Peirce's mental interests and imaginative powers, and this breadth characterized his teaching in Harvard College from beginning to end.

He was no teacher in the ordinary sense of that word. His method was that of the lecture or monologue, his students never being invited to become active themselves in the lecture room. He would stand on a platform raised two steps above the floor of the room, and chalk in hand cover the slates which filled the whole side of the room with figures, as he slowly passed along the platform; but his scanty talk was hardly addressed to the students who sat below trying to take notes of what he said and wrote on the slates. No question ever went out to the class, the majority of whom apprehended imperfectly what Professor Peirce was saying.

When I entered College in 1849 Professor Peirce had ceased to have to do with the elementary courses in Mathematics. He taught only students who had been through the two years of prescribed Mathematics and had elected to attend his courses, which were given three times a week throughout the junior and senior years. Two or three times in the course of the hour, Professor Peirce would stop for a moment or two to give opportunity for the members of the class to ask questions or seek explanations; and these opportunities were utilized by all the members who really wanted to learn. If a question interested him, he would praise the questioner, and answer it in a way, giving his own interpretation to the question. If he did not like the form of the student's question, or the manner in which it was asked, he would not answer it at all, but sometimes would address an admonition to the student himself which went home.

One day in my senior year, when Professor Peirce had already acquired the habit of giving me the highest possible marks on all my notes of his lectures and on every other exercise for which marks could be given, to the great concern of my competitor for the first place in the class, a concern which he liked to communicate to me his next door neighbor in Hollis—I graduated second—I ventured to say that what he had just been saying to us about functions and infinitesimal variables seemed to me to be theories or imaginations rather than facts or realities. Professor Peirce looked at me gravely, and remarked gently, "Eliot, your trouble is that your mind has a skeptical turn. Be on your guard against that tendency or it will hurt your career." That was new light to me; for I had never thought at all about my own turns of mind. The diagnosis was correct.

In spite of the defects of his method of teaching, Benjamin Peirce was a very

inspiring and stimulating teacher. He dealt with great subjects and pursued abstract themes before his students in a way they could not grasp or follow, but nevertheless filled them with admiration and reverence. His example was much more than his word. I remember that this great master began one day with unusual promptness to put on the slates a series of calculations and formulæ in which he seemed to be much interested. He said but little; but wrote diligently with the chalk, stopping now and then to examine his work and to rub out some of it, but only to resume it, and go on eagerly. The class before him said not a word, took notes as well as they could of what he wrote on the slates, and watched him. Suddenly near the end of the hour the worker looked despairingly at the contents of the last slate he had filled, turned to the class, and remarked, "there is an error somewhere in this work, but I cannot see where it is. This last line—the conclusion—is obviously wrong." Whereupon he seized the rubber and rapidly rubbed out everything he had put on the slates. Professor Peirce sat down in his armchair visibly fatigued. The class slowly folded their notebooks and departed without a word, even to each other. I, for one, have always remembered vividly that hour's spectacle.

In 1862, Thomas Hill was elected President of Harvard University. One of his first measures was the institution of courses of lectures open to graduate and other advanced students, and called University Lectures. President Hill's idea was to give advanced students living in Cambridge and the interested Cambridge public opportunities to hear the best scholars and scientists of the country speaking on their favorite subjects, the subjects in which they had won distinction or renown. Benjamin Peirce was one of the first persons to be appointed University lecturer, and he served gladly in this capacity in five different years. The University Lectures were not to be technical, though advanced. They were to be stimulating as well as informing, and women were encouraged to attend them as well as men. Benjamin Peirce's lectures dealt, to be sure, with the higher mathematics, but also with theories of the universe and the infinities in nature, and with man's power to deal with infinities and infinitesimals alike. His University Lectures were many a time way over the heads of his audience, but his aspect, his manner, and his whole personality held and delighted them. An intelligent Cambridge matron who had just come home from one of Professor Peirce's lectures was asked by her wondering family what she had got out of the lecture. "I could not understand much that he said; but it was *splendid*. The only thing I now remember in the whole lecture is this—'Incline the mind to an angle of 45°, and periodicity becomes non-periodicity and the ideal becomes real.'"

When Professor Bache retired from the superintendency of the U. S. Coast Survey, he procured the appointment of his intimate friend Benjamin Peirce as his successor in the superintendency. Those of us who had long known Professor Peirce heard of this action with amazement. We had never supposed that he had any business faculty whatever, or any liking for administrative work. A very important part of the Superintendent's function was to procure from

Committees of Congress appropriations adequate to support the varied activities of the Survey on sea and land. Within a few months it appeared that Benjamin Peirce persuaded Congressmen and Congressional Committees to vote much more money to the Coast Survey than they had ever voted before. This was a legitimate effect of Benjamin Peirce's personality, of his aspect, his speech, his obvious disinterestedness, and his conviction that the true greatness of nations grew out of their fostering of education, science, and art.

In his younger days Benjamin Peirce enjoyed taking part in private theatricals. As an actor he was apt to be too violent and impetuous; but he was always interesting. He had, indeed, a gift for dramatic expression which served him well in many incidents, both comical and tragical, of his maturer life. For this reason, among others, only persons who saw and heard him can fully appreciate the influence of his life and work.

II. Reminiscences.

By President A. Lawrence Lowell, Harvard University.

Looking back over the space of fifty years since I entered Harvard College, Benjamin Peirce still impresses me as having the most massive intellect with which I have ever come into close contact, and as being the most profoundly inspiring teacher that I ever had. His personal appearance, his powerful frame, and his majestic head seemed in harmony with his brain.

The amount of instruction in mathematics then given was small compared with what is offered in any large university at the present day. The teaching of the calculus and everything beyond was done by Benjamin Peirce and his son, the father at this period giving only the more advanced courses for the few upper classmen who elected them. He expected and received close and rapid attention in class, and hard, though not extensive, work outside. We read his *Analytic Mechanics*, Briot and Bouquet on *Elliptic Functions*, Tait and Hamilton on *Quaternions;* while his direct instruction consisted mainly, but not wholly, in solving problems by writing on the blackboard that covered the end of the room a series of equations which we copied into our notebooks.

As soon as he had finished the problem or filled the blackboard he would rub everything out and begin again. He was impatient of detail, and sometimes the result would not come out right; but instead of going over his work to find the error, he would rub it out, saying that he had made a mistake in a sign somewhere, and that we should find it when we went over our notes.

Described in this way it may seem strange that such a method of teaching should be inspiring; yet to us it was so in the highest degree. We were carried along by the rush of his thought, by the ease and grasp of his intellectual movement. The inspiration came, I think, partly from his treating us as highly competent pupils, capable of following his line of thought even through errors in transformations; partly from his rapid and graceful methods of proof, which reached a result with the least number of steps in the process, attaining thereby an artistic or literary character; and partly from the quality of his mind which

tended to regard any mathematical theorem as a particular case of some more comprehensive one, so that we were led onward to constantly enlarging truths. To those of us who have not pursued the study of mathematics since college days the substance of what he taught us has faded away, but the methods of thought, the attitude of mind and the mode of approach have remained precious possessions.

III. Reminiscences.

By Professor Emeritus W. E. Byerly, Harvard University.

When I entered Harvard in 1867, a particularly unsophisticated freshman from New Jersey knowing absolutely no one in the college, Cambridge was a small straggling town. The inhabitants still spoke of visiting Harvard Square as going down to the village.

The Square itself was occupied by the hay scales and the town pump. The portion of the college yard east of University Hall was a hayfield, from which the University drew a modest annual profit. The dormitories, Massachusetts, Hollis, Stoughton, and Holworthy, were grouped in the neighborhood of the college pump, the water supply of all the students in the yard. Steam heating and plumbing were unknown.

The College was small; the Faculty was small, but distinguished and picturesque. "There were giants in those days," bearded giants mainly, though Agassiz and Child were beardless, Sophocles, Longfellow, Lowell, Asa Gray, Benjamin Peirce. There are giants in the faculty now, but they are more or less lost in the crowd. Then, poets, discoverers, philosophers, and seers, in soft hats and long cloaks, looked their parts, and we newly-fledged freshmen gazed at them with admiration and awe.

The appearance of Professor Benjamin Peirce, whose long gray hair, straggling grizzled beard and unusually bright eyes sparkling under a soft felt hat, as he walked briskly but rather ungracefully across the college yard, fitted very well with the opinion current among us that we were looking upon a real live genius, who had a touch of the prophet in his make-up.

When I knew him later in the class-room, I will not say as a teacher, for he inspired rather than taught, and one's lecture notes on his courses were apt to be chaotic, I always had the feeling that his attitude toward his loved science was that of a devoted worshipper, rather than of a clear expounder. Although we could rarely follow him, we certainly sat up and took notice.

I can see him now at the blackboard, chalk in one hand and rubber in the other, writing rapidly and erasing recklessly, pausing every few minutes to face the class and comment earnestly, perhaps on the results of an elaborate calculation, perhaps on the greatness of the Creator, perhaps on the beauty and grandeur of Mathematics, always with a capital M. To him mathematics was not the handmaid of philosophy. It was not a humanly devised instrument of investigation, it was Philosophy itself, the divine revealer of TRUTH.

I remember his turning to us in the middle of a lecture on celestial mechanics and saying very impressively, "Gentlemen, as we study the universe we see every-

where the most tremendous manifestations of force. In our own experience we know of but one source of force, namely will. How then can we help regarding the forces we see in nature as due to the will of some omnipresent, omnipotent being? Gentlemen, there must be a GOD."

At another time he was lecturing on his favorite subject, which was then beginning to attract the attention of mathematicians and philosophers, Hamilton's new calculus of quaternions, which he believed was going to be developed into a most powerful instrument of research. He must have been working recently on his "Linear Algebras" for he said that "of possible quadruple algebras the one that had seemed to him by far the most beautiful and remarkable was practically identical with quaternions,[1] and that he thought it most interesting that a calculus which so strongly appealed to the human mind by its intrinsic beauty and symmetry should prove to be especially adapted to the study of natural phenomena. The mind of man and that of Nature's God must work in the same channels."

In one of his lectures on the theory of functions he established the relation connecting π, e, and i, $e^{\pi/2} = \sqrt[i]{i}$, which evidently had a strong hold on his imagination. He dropped his chalk and rubber, put his hands in his pockets, and after contemplating the formula a few minutes turned to his class and said very slowly and impressively, "Gentlemen, that is surely true, it is absolutely paradoxical, we can't understand it, and we don't know what it means, but we have proved it, and therefore we know it must be the truth."

I have hinted that his lectures were not easy to follow. They were never carefully prepared. The work with which he rapidly covered the blackboard was very illegible, marred with frequent erasures, and not infrequent mistakes (he worked too fast for accuracy). He was always ready to digress from the straight path and explore some sidetrack that had suddenly attracted his attention, but which was likely to have led nowhere when the college bell announced the close of the hour and we filed out, leaving him abstractedly staring at his work, still with chalk and eraser in his hands, entirely oblivious of his departing class.

Outside of the class-room I used to see him at meetings of a little informal mathematical club, attended by the more advanced students, where he frequently took part in the discussions and was always alert and suggestive; and at meetings of the American Academy where he frequently took an active part in the informal debate on the paper of the evening, usually to the enlightenment or the discomfiture of the author.

The first meeting of the Academy I ever attended gave him an opportunity to show his remarkable ability to think clearly and quickly. The paper of the evening was a very elaborate one, describing the lecturer's investigations into the tides of The Gulf of Maine. An important member of the Coast Survey, he had been engaged all summer in hydrographic work at the mouth of the Bay of Fundy, but he confessed himself completely staggered by the phenomena he had

[1] Compare pages 15–16, 28 of this monograph.—R. C. A.

observed and had just described to us, which seemed to him absolutely inexplicable. At the close of the address Professor Peirce rose from his seat and began to ask leading questions. The lecturer, rather puzzled at first, began to answer them hesitatingly but soon discovered that step by step he was being led up to a theory that met all his difficulties and dissolved all his paradoxes. It was as pretty a piece of work as ever I saw done, and was manifestly entirely unrehearsed.

Benjamin Peirce, mathematician and mystic, was not always on the heights. Calling at his house one day to consult him on some abstruse problem I found him on all fours in the parlor playing bear with one of his grandchildren, and I was invited to take part in the game.[1]

In his personal relations with his students he was always courteous, kind, and helpful, if rather prone to overrate their ability and promise, and they reverenced and loved him.

IV. Reminiscences.

By Chancellor Arnold B. Chace, Brown University.

I have very pleasant memories of Professor Benjamin Peirce. In the later seventies being desirous of taking up the study of quaternions, which were then beginning to be talked about, and having worked at them myself for a while, I decided that I needed some help—and, going one day to Cambridge, after making some inquiries, I called on Professor Peirce, introduced myself, and asked him if he would assist me. He received me very pleasantly and seemed much interested in my request. He had at that time retired from active service in the college, but, as he told me with reference to a recent request from the head of Radcliffe, which was just starting, he was glad to assist anyone who deserved it.

I went to his house one afternoon a week for nearly a year and, sitting in his pleasant study before an open fire, I would show him the work that I had done in the previous week, and he, an old man, and I, a young man, discussed quaternions and many other matters in a most friendly way.

He was one of the most stimulating men that I have ever known. I can picture him now with his large noble brow, his beautiful white hair, his flashing eyes, his animated but kindly face, and most inspiring personality.

[1] A quotation from Henry Cabot Lodge's *Early Memories* (New York, 1913, pp. 55–56) may be recalled in this connection: "Altogether he had a fascination which even a child felt, and all the more because he was full of humor, with an abounding love of nonsense, one of the best of human possessions in this vale of tears. I know that I was always delighted to see him, because he was so gentle, so kind, so full of jokes with me, and so 'funny.' As time went on I came as a man to know him well and to value him more justly, but the love of the child, and the sense of fascination which the child felt, only grew with the years."

Another germane quotation may be made from E. W. Emerson's *The Early Years of the Saturday Club* (Boston and New York, 1918, p. 102): "A pleasant reminiscence of the family life is given by his daughter, another instance of Leasts and Mosts in this remarkable man. Before breakfast he always went to walk with his younger children, now a delightful memory to them. This man who could divine and see remotest suns in space, amused his little ones by allowing no pin to hide from his eyes in the dust of the sidewalk;—'although he never seemed to be looking for them, he would suddenly stoop to pick up a pin. He had various 'pincushions'; one was the trunk of an elm tree near our gate, others on Harvard and Brattle streets. Those on Quincy and Kirkland streets are still standing.' "—R. C. A.

He was very enthusiastic in his belief that Hamilton's *Lectures on Quaternions* and *Elements of Quaternions* marked a most important step in the progress of mathematical science, a belief which I think has been fully justified in all our modern vector analysis. A recent writer has placed [1] these books of Hamilton's among the ten most important works in the development of mathematics.

It was at just about this date, in the winter 1878–79, that Professor Peirce delivered a course of lectures at the Lowell Institute entitled "Ideality of the Physical Sciences." In the sixth and last of these lectures he made the statement that "Ideality is preëminently the foundation of mathematics." It was in this lecture also, after stating that two geometers had computed independently of each other the elements of the orbit of a planet which would reconcile the apparent discrepancies in the orbit of the recently discovered planet Uranus, and had determined that this new planet was situated at a certain point in the heavens, he told how on December 23, 1846, Dr. Galle of Berlin directed his telescope at the designated spot and discovered the new planet Neptune. Professor Peirce then made the surprising statement that the planet so discovered by Dr. Galle was not in the place that had been figured out, but that there were two possible positions of the planet and that by a remarkable coincidence on the given night the two positions were in a straight line from the earth. I remember very well as I sat in his study that he repeated this story to me with much animation, and when I questioned him further about it he said he was sure of his reasoning, but the calculations were so long and laborious that he had never had the courage to go through them a second time.[2]

V. Biographical Sketch.[3]

By R. C. Archibald, Brown University.

Mathematical research in American Universities began with Benjamin Peirce. His influence on students and contemporaries was extraordinary; this is borne out by the "Reminiscences" given above. In September, 1924, President Lowell wrote also: "I have never admired the intellect of any man as much as that of Benjamin Peirce. I took every course that he gave when I was in College, and whatever I have been able to do intellectually has been due to his teaching more than to anything else."

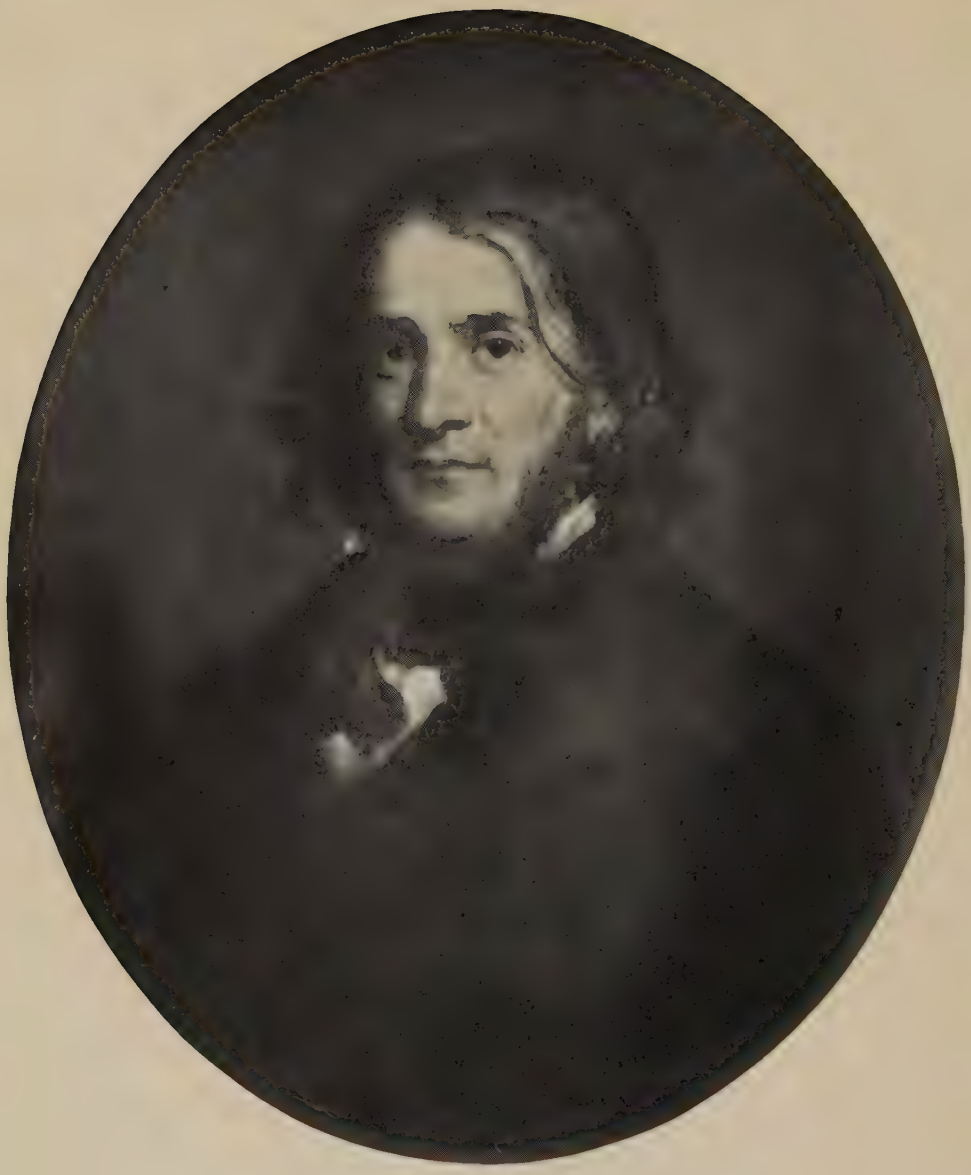
Hence no apology is required for taking the greater part of an issue of the Monthly to exhibit the life and work of such a man, if information of this kind is not already easily available. An appreciative sketch appeared in this Monthly nearly thirty years ago,[4] but it seemed evident that something of a more com-

[1] In this Monthly, *1923*, 320.—R. C. A.

[2] Compare page 14 of this monograph.—R. C. A.

[3] The gist of this section and of the next was given in a paper on Benjamin Peirce read at a joint session of the History of Science Society, Section L of the American Association for the Advancement of Science, and of the Mathematical Association of America, Washington, D. C., January 1, 1925.

[4] "Benjamin Peirce" by F. B. Matz, in this Monthly, *1895*, 173–179; also in *A Mathematical Solution Book* by B. F. Finkel, fourth ed., Springfield, Mo., 1902, pp. 524–528.



BENJAMIN PEIRCE, 1853 (?)

From a painting by J. A. Ames in the possession of Harvard University.

prehensive nature should be attempted, and that sources of further information should be indicated for the present generation and for the future historian of American mathematics. Previously [1] there has been no adequate indication of the extent of Peirce's publications; even the lists of his periodical articles in the *Royal Society Catalogue of Scientific Papers*, and in "Poggendorff," [2] are by no means complete. Furthermore, critical estimates of Peirce's notable work in linear associative algebra, in connection with the problem of the discovery of Neptune, and in other fields, are not readily to be found by the average inquirer.

Benjamin Peirce was born at Salem, Mass., April 4, 1809, and died at Cambridge, Mass., October 6, 1880. He was descended from John Pers, a weaver of Norwich, Norfolk Co., England, who emigrated to this country in 1637. His father was Benjamin Peirce (1778–1831), who graduated from Harvard University in 1801, represented Salem in the lower branch of the legislature for several years and was later sent to the state senate. For the last five years of his life he was librarian at Harvard University; his history of the University was published after his death.

Benjamin Peirce entered Harvard University in 1825 and graduated in the class of 1829, with membership in the Phi Beta Kappa Society. Oliver Wendell Holmes, James Freeman Clarke, educator and prolific author, and George T. Bigelow and Benjamin R. Curtis, eminent jurists, were classmates. For the two years immediately after graduation, Peirce was associated with George Bancroft as teacher at the famous Round Hill School, Northampton, Mass. In 1831 he was appointed tutor in mathematics at Harvard College and

[1] Some of the best printed sources of information concerning Peirce's ancestry, life and work are: (1) F. C. Peirce, *Peirce Genealogy*, Worcester, 1880; (2) R. S. Rantoul, *Historical Colls. Essex Institute*, vol. 18, 1881, pp. 161–176; (3) *Proc. Amer. Acad. Arts and Sciences*, vol. 16, 1881, pp. 443–454, by [H. A. Newton]. This appeared in slightly different form in *Amer. Jl. Sci.*, vol. 122, 1881, pp. 167–178; (4) *Benjamin Peirce . . . A Memorial Collection*, edited by M. King, Cambridge, Mass., 1881, 64 pp. [includes sketch by Thomas Hill from *The Harvard Register*, May, 1880; editorials and sketches from various newspapers and periodicals; sermons by A. P. Peabody, T. Hill and C. A. Bartol; and poems by Oliver Wendell Holmes, G. Thwing and T. W. Parsons. The poem by Holmes appeared originally in the *Atlantic Monthly*, vol. 46, 1880, p. 823; also in *Kansas City Review*, vol. 4, p. 510]; (5) *Mo. Notices Royal Astr. Soc.*, vol. 41, 1881, pp. 191–193; (6) *Proc. Royal Soc. Edinb.*, vol. 22, 1882, pp. 739–743, by Simon Newcomb; (7) "The Services of Benjamin Peirce to American mathematics and astronomy" by J. J. See, *Popular Astronomy*, vol. 3, 1895, pp. 49–57; (8) *Encyclopædia Britannica*, eleventh edition, vol. 11, 1911; (9) F. Cajori, *A History of Mathematics*, second ed., 1919, pp. 338–339, etc.; and (10) E. W. Emerson, *The Early Years of the Saturday Club, 1855–1870*, Boston and New York, 1918, pp. 96–109. Other references will be given in the following pages. Sketches of minor importance occur in: (1) F. S. Drake, *Dictionary of American Biography*, Boston, 1872; (2) *The Harvard Book, a series of historical and biographical and descriptive sketches* by various authors collected and publ. by F. O. Vaille and H. A. Clark, vol. 1, Cambridge, 1875, pp. 104, 172–173; (3) Appleton's *Cyclopædia of American Biography*, New York, vol. 3, 1888; and (4) *National Cyclopædia of American Biography*, New York, vols. 8, 9, 10, 1898, 1907, 1909.

A considerable quantity of Benjamin Peirce's manuscripts and correspondence was presented to the American Academy of Arts and Sciences in 1913. This collection is soon to be augmented by many other letters of great value which have been in the possession of Peirce's grandson, Benjamin P. Ellis of Cambridge, Mass. I am much indebted to Mr. Ellis for allowing me free access to this material.

[2] J. C. Poggendorff, *Biographisch-Literarisches Handwörterbuch zur Geschichte der exacten Wissenschaften*, Leipzig, vols. 2, 3, 1863, 1898.

was in full charge of the mathematical work. In 1833 he received the A.M. degree from Harvard, was appointed professor of mathematics and natural philosophy, and was married to Sarah H. Mills of Northampton. His academic title was changed to that of professor of astronomy and mathematics in 1842.

Professor Peirce had four sons and a daughter. One son, Benjamin Mills (1844–70), was a mining engineer; another, Herbert Henry Davis [1] (1849–1916), was a diplomat.[2] The eldest, James Mills (1834–1906), was assistant professor of mathematics at Harvard 1861–69, and professor from 1869 till his death; he was also dean of the graduate school of Arts and Sciences 1890–95, and of the faculty of Arts and Sciences 1895–98. But the son who seemed largely to inherit his father's intellectual powers [3] was Charles Santiago Saunders (1839–1914) who was lecturer at Harvard, in philosophy and logic, 1869–71. Of these brothers, the brilliant Benjamin Osgood Peirce (1854–1914) was a second cousin once removed.[4]

Professor Benjamin Peirce was honored in various ways both in this and in other countries. He was elected a Member of the American Philosophical Society in 1842; an Associate of the Royal Astronomical Society, London, 1850; a Foreign Member (limited to 50) of the Royal Society of London in 1852; an Honorary Member of the State Historical Society of Wisconsin, 1854; a Fellow of the American Academy of Arts and Sciences in 1858; an Honorary Fellow of the University of St. Vladimir, at Kiev, Russia (now Ukrainia), 1860; a Corresponding Member of the British Association for the Advancement of Sciences in 1861; an Honorary Fellow (limited to 36) of the Royal Society of Edinburgh in 1867; and a Correspondent in the mathematical class of the Royal Society of Sciences at Göttingen in 1867. He received the degree of LL.D. from the University of North Carolina in 1847, and from Harvard [5] in 1867. He was: One of a committee of five appointed by the American Academy of Arts and Sciences to draw up a "Program for the Organization of the Smithsonian Institution," 1847; Consulting Astronomer for the Nautical Almanac, 1849–67; Director of the

[1] Probably named after Admiral Charles Henry Davis (1807–77), who married Benjamin Peirce's wife's sister.

[2] See *National Cyclopædia of American Biography*, vol. 10, p. 449, and vol. 9, p. 539.

[3] Papers of C. S. S. Peirce are referred to in such works as: B. Russell, *The Principles of Mathematics*, vol. 1, Cambridge, 1903; C. I. Lewis, *A Survey of Symbolic Logic*, Berkeley, 1918; and F. Enriques, *Per la Storia della Logica*, Bologna, 1922. See, also, E. W. Davis, "Charles Peirce at Johns Hopkins," *The Mid-West Quarterly*, New York, vol. 2, pp. 48–56; it is here stated that Sylvester considered C. S. S. Peirce "a far greater mathematician than his father."

[4] At Harvard he was instructor in mathematics, 1881–84, then assistant professor of mathematics and physics, 1884–88, and finally professor of mathematics and natural philosophy, 1888–1914.

[5] In a letter dated July 29, 1867, the president of Harvard College, who was also somewhat of a mathematician, wrote as follows:

"I have the honor of informing you that the University, on Commencement Day, conferred upon you the degree of Doctor of Laws in recognition of the transcendent ability with which you have pursued mathematical physical investigations, and in particular for the luster which she has herself for so many years borrowed from your genius.

"With the sincerest regard,

"Very truly and gratefully yours,

"Thomas Hill."

longitude determinations of the United States Coast Survey, 1852–67; Member of the Scientific Council (J. Henry, A. D. Bache, B. Peirce) of the Dudley Observatory, Albany, 1855–58; Superintendent of the Coast Survey, February 26, 1867, to February 16, 1874, while continuing to serve as professor at Harvard; consulting geometer of the Survey,[1] 1874–80; President of the American Association for the Advancement of Science, 1853, and elected Fellow, 1875; Chairman of the Department of Education of the American Social Science Association, 1869–72, acting president [2] in 1878, and vice-president in 1880; One of the fifty incorporators of the National Academy of Sciences, one of the nine members of the committee of organization, and chairman of the mathematics and physics class,[3] 1863; Director of the expedition to Sicily to observe the eclipse of the sun, December, 1870; Coöperating Editor of the *American Journal of Mathematics*, volume 1, 1878; Special Lecturer on physical philosophy at the Concord Summer School of Philosophy and Literature, 1879; Lecturer at the Lowell Institute, 1879, and at the Peabody Institute, 1880.

In a recently published article, Professor Coolidge pointed out [4] that before the time of Benjamin Peirce it never occurred to anyone that mathematical research "was one of the things for which a mathematical department existed. Today it is a commonplace in all the leading universities. Peirce stood alone—a mountain peak whose absolute height might be hard to measure, but which towered above all the surrounding country." In his publications [5] and papers before scientific bodies, Peirce touched on a wide range of topics.

Of his eleven works, in twelve volumes, six were elementary texts, some of which went through several editions. The first, on plane trigonometry, appeared in 1835, and the second and third, on spherical trigonometry and sound, in 1836. The seventh work, in two volumes (1841–46), dealt with analytic geometry,

[1] This post was held while retaining his professorship. He was appointed "consulting geometer" with compensation at the rate of $4,000 per annum, and subsistence at the same rate per diem as was allowed the late Hydrographic Inspector." For an account of Peirce's connection with the Survey, see *Centennial Celebration of the United States Coast and Geodetic Survey, April 5 and 6, 1916*, Washington, 1916, p. 137. T. C. Mendenhall here remarks, "As a genius in mathematics and astronomy he is easily a star of first magnitude in the Coast Survey galaxy." In an account of the centennial celebration in the *Scientific Monthly*, vol. 3, 1916, p. 616, the story is told that at a meeting of the National Academy of Sciences he spent an hour filling the blackboard with equations, and then remarked, "There is only one member of the Academy who can understand my work and he is in South America." Was this B. A. Gould? Hilgard was the managing head of the Survey during Peirce's administration; see anonymous note by S. Newcomb, *Nation*, March 5, 1874, vol. 13, p. 157.

[2] He declined to take the titular office of president offered to him in this year although performing all the duties of the office. For a sketch of Professor Peirce including an account, by F. B. Sanborn, of his relations to the American Social Science Association, see *Journal of Social Science*, no. 12, 1880, pp. ix–xi. For the title of his address delivered before the Association in 1878, see the List of Peirce's Writings in the next Section.

[3] Compare *A History of the First Half-Century of the National Academy of Sciences, 1863–1913*, Washington, 1913, pp. 9, 10, 20, 21, 23, 27, 168–171, 215, 223, and 256. Peirce was one of the first sixteen to read papers before the Academy, January, 1864.

[4] "The Story of Mathematics at Harvard" by J. L. Coolidge, *Harvard Alumni Bulletin*, January 3, 1924, vol. 26, p. 376.

[5] A list of these, which probably closely approximates to completeness, is given in Section VI of this monograph.

differential and integral caculus, and differential equations. His other volumes were: *Tables of the Moon* (1853–56); a notable work on *Analytic Mechanics* (1855), the remarkably original *Linear Associative Algebra* (1870), and the posthumous volume of lectures, *Ideality in the Physical Sciences* (1881).

About one quarter of the titles of Peirce's publications relate to topics of pure mathematics and three quarters to questions mainly in the fields of astronomy, geodesy and mechanics. His first publications, when only sixteen years of age, were solutions of problems in algebra and mechanics. Very early in life, possibly through having Ingersoll Bowditch as schoolmate, Peirce had the good fortune to become acquainted with Ingersoll's father, Dr. Nathaniel Bowditch [1] (1773–1838), author of *The New American Practical Navigator* (which has gone through so many editions in the last hundred years), and translator of Laplace's celestial mechanics. During the ten years before he was thirty, Peirce revised and corrected the proof sheets of this translation. Among other works, Peirce contributed an original notable result regarding perfect numbers; gave certain methods of determining the number of real roots of equations applicable to transcendental as well as to algebraic equations; made an important advance in the treatment of Kirkman's school-girl problem; [2] discussed a new binary system of arithmetic; wrote on probabilities at the three-ball game of billiards, on the extension of Lagrange's theorem for development of functions, and on transformation of curves; and supplemented his volume on associative algebra by a memoir on the uses and transformations of linear algebras. Apart from volumes already referred to, his publications on applied mathematics included: various papers on the perturbations of Neptune and Uranus; a mathematical treatment (also translated into German) of the problem of Saturn's rings, leading to the result that the rings were fluid; [3] a note upon the conical pendulum; papers on the relation between the elastic curve and the motion of the pendulum, on a criterion for the rejection of doubtful observations, on the catenary on a vertical right cone, on the internal constitution of the earth, and on a mathematical investigation of the fractions which occur in phyllotaxis.

The concluding characteristic paragraph of this last paper is as follows:

"May I close with the remark, that the object of geometry in all its measuring and computing, is to ascertain with exactness the plan of the great Geometer, to penetrate the veil of material forms, and disclose the thoughts which lie beneath them? When our researches are successful, and when a generous and heaven-eyed inspiration has elevated us above humanity, and raised us triumphantly into the very presence, as it were, of the divine intellect, how instantly and entirely are human pride and vanity repressed, and, by a single glance at the glories of the infinite mind, are we humbled to the dust." [4]

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[1] The dedication of Peirce's *Analytic Mechanics* is as follows: "To the cherished and revered memory of my master in science, Nathaniel Bowditch, the father of American geometry, this volume is inscribed."

[2] Sylvester referred to this treatment as "the latest and probably the best" (*Philosophical Magazine*, 1861, vol. 21, p. 520; also *Collected Mathematical Papers of . . . Sylvester*, vol. 2, 1908, p. 276). See also my notes on this title in the next Section.

[3] Compare S. Newcomb, *Popular Astronomy*, 1879, p. 358; also Newcomb-Engelmann, *Populäre Astronomie*, 5th ed., 1914, p. 429.

[4] Another quotation may be given to illustrate Peirce's manner of thought to which reference has been made above in the "Reminiscences." The following are concluding sentences from a

While Peirce read before scientific societies many papers concerning his investigations, the printed reports of them are unfortunately often mere abstracts. "His mind moved with great rapidity, and it was with difficulty that he brought himself to write out even the briefest record of its excursions."

"His elementary books were remarkable for their condensation. In the geometry, especially, the short and terse and comprehensive forms of mathematical thought and expression, natural to the mathematician, were substituted for the minute demonstrations of Euclid. Free use was also made of infinitesimals." [1]

In order to bring out more clearly the place Peirce occupies in the development of American mathematics it seems desirable to comment further on four of the subjects which he discussed in a notable manner: 1. criterion for the rejection of doubtful observations; 2. perturbations of Uranus and the discovery of Neptune; 3. analytic mechanics; 4. linear associate algebra.

1. "Peirce's criterion," as the term has appeared in scientific literature, had as its object the solution of a delicate and practically important problem of probability; this problem is: "Being given certain observations of which the greater part is to be regarded as normal and subject to the ordinary law of error adopted in the method of least squares, while a smaller unknown portion is abnormal, and subject to some obscure source of error, to ascertain the most probable hypothesis as to the partition of the observations into normal and abnormal." This criterion has been regarded as one of Peirce's best contributions to science. In volume 46 (St. Petersburg, 1898) of the great Russian encyclopædia (based on "Brockhaus") it is especially referred to in a ten-line biographical notice of Peirce.

The excessive brevity of Peirce's statement concerning the criterion, when it appeared in 1852, resulted in frequent misunderstandings. The tables which Gould published three years later [2] facilitated its application. But it was not till 1878 that Peirce somewhat remedied his original statement by giving fuller explanations. In 1920, however, R. M. Stewart proved the statement fallacious.[3]

The criterion and its application are set forth at length in W. Chauvenet's *Manual of Spherical and Practical Astronomy* [4] (1868), and a paragraph is devoted to it in W. S. Jevons's *Principles of Science* [5] (1877). An illustration of a recent work where application of the criterion is suggested is H. M. Wilson's *Topographic, Trigonometric and Geodetic Surveying* [6] (1912).

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paper on Saturn's rings: "But in approaching the forbidden limits of human knowledge, it is becoming to tread with caution and circumspection. Man's speculations should be subdued from all rashness and extravagance in the immediate presence of the Creator. And a wise philosophy will beware lest it strengthen the arms of atheism, by venturing too boldly into so remote and obscure a field of speculation as that of the mode of creation which was adopted by the Divine Geometer."

[1] S. Newcomb, *Royal Soc. Edinb., Proc.*, vol. 22, p. 739.

[2] B. A. Gould, "On Peirce's criterion for the rejection of doubtful observations with tables for facilitating its application," *Astron. Jl.*, vol. 4, pp. 81–87. G. B. Airy expressed himself as believing Peirce's criterion defective in its foundation and illusory in its results (*Astron. Jl.*, vol. 4, pp. 137–138, 1856); Joseph Winlock showed (pp. 145–146) that his argument was wholly unsound. Compare articles by Stone and Glaisher in *Mo. Notices R. Astr. Soc.*, vols. 28, 33–35.

[3] *Popular Astronomy*, vol. 28, pp. 2–3. See also J. L. Coolidge, *An Introduction to Mathematical Probability*, Oxford, 1925, pp. 126–127.

[4] Volume 2, fourth edition, 1868, pp. 558–566, 596–599.

[5] London, second edition, p. 391.

[6] New York, third edition, pp. 604–606.

2. The computation of the general perturbations of Uranus and Neptune was the first work to extend Peirce's reputation. Simon Newcomb's compact statement [1] in this connection is here reproduced with two added footnotes:

"The formulæ to which he was led were published in the first volume of the *Proceedings of the American Academy*,[2] but were accompanied by no description of his process. Subsequent investigations, however, showed them to have been remarkably accurate. In his views of the discrepancy between the mean distance of Neptune as predicted by Leverrier, and as deduced from observations, he was less fortunate, although when due consideration is given to Leverrier's conclusions, there was much plausibility in the position taken by Peirce. As the subject has frequently been discussed without a due comprehension of all the circumstances, a brief review of them may be appropriate.[3]

"Leverrier, from his researches, found for the mean distance of the disturbing planet, 36.1539, and a consequent period of 217 years. He also announced that the limits of the mean distance which would satisfy the observed perturbation of Uranus were 35.04 and 37.90. He founded this conclusion on a supposed inadmissible increase of the outstanding differences between theory and observation, as the mean distance was diminished below 35. But when the planet was discovered, its mean distance was found to be only 30; and yet the observations of Uranus were as well satisfied as by Leverrier's hypothetical planet. It was, therefore, an expression of Peirce's high confidence in the accuracy of Leverrier's conclusions that led him to announce that there were two solutions to the problem; the one being that found by Leverrier, and the other that corresponding to the actual case. He also sought to show a cause for the two solutions in a supposed discontinuity in the form of the perturbations, when the period was brought to the point at which five revolutions of Uranus would be equal to two of Neptune. As a matter of fact, however, it has been shown by Professor Adams that there is no such discontinuity in the actual perturbations during the limited period; from which it would follow that Leverrier must have made a mistake in tracing out the conclusions which would follow when the mean distance of the disturbing planet was diminished."

H. H. Turner has also published a very readable account of this matter in his *Astronomical Discovery* (London, 1904).

3. As to Peirce's *Analytic Mechanics*, Simon Newcomb referred to it [4] as "the most characteristic as well as the most extensive of his works." Then he continues: "The exposition of dynamical concepts in the first forty pages is pleasant reading for one already acquainted with the subject, but that a student beginning the subject could understand it without clearer distinction of definitions, axioms, and theorems seems hardly possible." In his later years Peirce often said he wanted to rewrite his *Mechanics* and introduce quaternions into it. Sir

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[1] *Roy. Soc. Edinb., Proc.*, vol. 22, pp. 740–741.

[2] Published in 1848. It is related that when, in 1846, Peirce announced in the American Academy that Galle's discovery of Neptune in the place predicted by Leverrier was a happy accident, the President, Edward Everett, "hoped the announcement would not be made public: nothing could be more improbable than such a coincidence."—"Yes," replied Peirce, "but it would be still more strange if there was an error in my calculations,"—a confident assertion which the lapse of time has vindicated. In this connection, it is noteworthy that Peirce was not made a fellow of the American Academy till many years after he had been honored by the American Philosophical Society and two foreign bodies. In 1878 Peirce sent in his resignation as a Fellow of the Academy, but this was never accepted.

[3] A full statement of Peirce's views, which he maintained to the last, is given by J. M. Peirce, on pages 200–211 of B. Peirce's *Ideality in the Physical Sciences*. It was 28 years after Peirce's criticisms of the work of Leverrier and Adams were published (1848), that is, 1876, that J. C. Adams made a reply, *Journal de Mathématiques* (Liouville), vol. 41, 1876; see also *The Scientific Papers of John Couch Adams*, vol. 1, 1896, pp. xxxiii, 57, 64.

[4] *Royal Society Edinb., Proc.*, vol. 22, p. 742.

Thomas Muir has given [1] an analysis of the section on determinants and functional determinants. The analysis is as follows:

"At the outset of his Tenth Chapter, which deals with the integration of the differential equations of motion, Peirce feels the need for making his reader acquainted with the properties of functional determinants. He accordingly gives as a preparation a brief account (§§ 327–348, pp. 172–183) of determinants in general, and then expounds within the space of sixteen broad-margined pages the main theorems of Jacobi's 'De determinantibus functionalibus.' The treatment of the original is free and masterly, the order being altered with good effect. For example, Jacobi's incorrectly stated proposition is brought forward to occupy the second place, the enunciation being *If either* (i.e., *any one*) *of the given functions contains any of the other functions, these* (*latter*) *functions may be regarded as constant in finding the functional determinant.* There is thence deduced Jacobi's last proposition of all, namely, that expressing the determinant as a single product: and this in turn is used to discuss the connection between the vanishing of the determinant and interdependence of the functions.

"Had Peirce's exposition been less condensed and been published as part of an ordinary text-book of determinants, its value at this time to English-speaking students would have been considerable."

4. There seems to be no question that his *Linear Associative Algebra* [2] was the most original and able mathematical contribution which Peirce made. He himself held the work in high esteem; on April 4, 1870, he wrote in the introduction, "This work has been the pleasantest mathematical effort of my life. In no other have I seemed to myself to have received so full a reward for my mental labor in the novelty and breadth of the results." In his *Synopsis of Linear Associative Algebra* (published by the Carnegie Institution in 1907) J. B. Shaw characterized the work as "really epoch-making," and devoted a number of pages (52–55, 101–106) to formulating the main results. While the monograph attracted wide and favorable comment in England and America,[3] continental investigators on the subject (1889–1902) did not give Peirce the credit which his results and methods deserved. Adverse criticism had been "due in part to a misunderstanding of Peirce's definitions, in part to the fact that certain of Peirce's principles of classification are entirely arbitrary and quite distinct in statement from those used by Study and Scheffers,[4] in part to Peirce's vague and in some cases unsatisfactory proofs, and finally to the extreme generality of the point of view from which his memoir sprang, namely a 'philosophic study of the laws of algebraic operation.'" In order that Peirce's work should receive due recognition, H. E. Hawkes discussed and answered the following questions: [5]

[1] T. Muir, *The Theory of Determinants*, London, vol. 2, 1911, p. 251.

[2] The first sentence of the work is often quoted. It is: "Mathematics is the science which draws necessary conclusions." On page 5 we find a reference to the "mysterious formula" $i^{-i} = e^{\pi/2} = 4.810477381$. Compare this MONTHLY, *1921*, 115–121.

[3] The substance of the work was reviewed by William Spottiswoode in his retiring presidential address delivered before the London Mathematical Society in 1872 ("Remarks on Some Recent Generalizations in Algebra," *London, Math. Soc., Proc.*, vol. 4, 1873, pp. 147–164). Peirce refers to this (1875) as a "fine, generous, and complete analysis." See also Cayley, *Collected Mathematical Papers*, vol. 11, pp. 457–8; vol. 12, pp. 60–71, 106, 303, 459, 465. The first reference is to Cayley's address before the British Association in 1883, when he spoke of "the valuable memoir by the late Benjamin Peirce." At the last reference Cayley writes (1887): "the general theory of associative linear forms is treated in a very satisfactory manner in Peirce's memoir."

[4] A complete list of references may be found in Shaw's work. So also for references to other works developed from Peirce's ideas.

[5] "Estimate of Peirce's Linear Associative Algebra," *Amer. Jl. Math.*, vol. 24, 1902, pp. 87–95; "On Hypercomplex Number Systems," *Amer. Math. Soc., Trans.*, vol. 3, 1902, pp. 312–330.

(1) What problem did Peirce attack, and to what extent did he solve it? (2) What relation does this problem bear to that treated by Study and Scheffers? (3) To what extent do Peirce's methods assist in the solution of that problem? In part Hawkes summed up his conclusions as follows:

"We can now state precisely the problem that Peirce set for himself. He aimed to develop so much of the theory of hyper-complex numbers as would enable him to enumerate all inequivalent, pure, non-reciprocal number systems in less than seven units. The relation to the problem treated by Scheffers is plain if we remember that the first two of Peirce's principles of classification are identical with those of Scheffers, and the other three are only slightly modified. Peirce solved this problem completely. The theorems stated by him are in every case true, though in some cases his proofs are invalid."

Hawkes showed also that by using Peirce's principles as a foundation, we can deduce a "method more powerful than those hitherto given," by such writers as Study and Scheffers, for enumerating all number systems of the type considered by Scheffers. Since Study is the author of the article on "Theorie der gemeinen und höheren complexen Grössen" in the *Encyklopädie der math. Wissenschaften*, one is not surprised to find his references to Peirce so wholly inadequate.[1]

Benjamin Peirce died in 1880 in the seventy-second year of his life and in the fiftieth of his service in the University. The teaching of mathematics at Harvard during this half century has been described by Florian Cajori[2] and J. K. Whittemore.[3] Other information of interest, supplementing the "Reminiscences" given above, may be found in the printed Reports of the presidents of the University during those years, in the 1829 Class Records (Harvard Univ. Library), and in the first volume of the *Annals of the Harvard Observatory*.

In the account of "How I was Educated," Edward Everett Hale, '39, wrote:[4] "The classical men made us hate Latin and Greek; but the mathematical men (such men! Pierce [*sic*] and Lovering) made us love mathematics, and we shall always be grateful to them." Lovering taught mathematics and natural philosophy at Harvard, 1838–88. In another place Hale wrote:[5] "I had but four teachers in college,—Channing, Longfellow, Peirce and Bachi. The rest heard me recite but taught me nothing."

Colonel Henry Lee has written of Peirce as follows in *The Harvard Book*:[6]

"Why we should have given him the affectionate diminutive name of 'Benny' I cannot say, unless as a mark of endearment because he could fling the iron bar upon the Delta farther than any undergraduate, or perhaps because he always thought the bonfire or disturbance outside the college

[1] Volume 1, pp. 159 and 167. In Cartan's form of the article, *Encyclopédie des Sciences Mathématiques*, tome 1, vol. 1, a very different presentation is found; see, for example, pp. 369, 401–2, 417, 422–5. While scores of trivialities are reviewed in *Jahrbuch über die Fortschritte der Mathematik* for 1881, absolutely nothing is given concerning Peirce's notable work.

[2] F. Cajori, *The Teaching and History of Mathematics in the United States*, Washington, 1890, pp. 133–148, 278, 397. See also J. L. Coolidge, Story of math. at Harvard, *l. c.*, pp. 372–376.

[3] In a sympathetic sketch of "James Mills Peirce," *Science*, n.s., vol. 24, 1906, pp. 40–48. Rather curiously J. M. Peirce, who died in 1906, was, as his father also, in the seventy-second year of his life and in the fiftieth of his service in the University.

[4] *Forum*, vol. 1, 1886, p. 61.

[5] *Outlook*, June 4, 1889, vol. 59, p. 316; also in E. E. Hale, *James Russell Lowell and his Friends*, Boston, 1899, p. 128.

[6] Volume 1, Cambridge, 1875, p. 104.

grounds, and not inside, and conducted himself accordingly. His softly lisped *sufficient* brought the blunderer down from the blackboard with a consciousness of failure as overwhelming as the severest reprimand. There was a delightful abstraction about this absorbed mathematician which endeared him to the students, who hate and torment an instructor always on the watch for offences, and which confirmed the belief in his peculiar genius."

Peirce's force and judgment in a great emergency are shown in the following anecdote by one who was present:[1]

Jenny Lind's last concert of the original series, given under the auspices of Phineas T. Barnum, was given at the hall over the Fitchburg Railroad Station. Tickets were sold without limit,—many more than the hall could hold,—and there was every prospect of a riot. Barnum had taken the precaution to leave for New York. I got about one-third up the main aisle, but could get no farther. Just ahead of me was Professor Peirce. The alarm was increasing. The floor seemed to have no support underneath, but to hang over the railroad track by steel braces from the rafters above. Would it hold? The air was stifling and windows were broken, with much noisy crashing of glass, in order to get breath. Women were getting uneasy. And there was no possibility of escape from a mass of human beings so packed together. We knew, from the conductor's baton, that the orchestra was playing, but no musical sound reached us. Professor Peirce mounted a chair. Perfect silence ensued as soon as he made himself seen. He stated, very calmly, certain views at which he had arrived after a careful study of the situation. The trouble was at once allayed. Jenny Lind recovered her voice and the concert went on to its conclusion."

Another contemporary has made this record:[2]

"He was among the first to read any new and noteworthy poem [3] or tale, to hear a new opera or oratorio; and his judgment and criticism upon such matters was keen and original. His interest in religious themes was deep, but it was in the fundamental doctrines rather than in the debates of sectarians; he was a devout believer in Christianity, but held to no established creed."

Among Peirce's students who afterwards became eminent were Simon Newcomb and George W. Hill. In his *Reminiscences of an Astronomer*,[4] Newcomb has happily hit off some of Peirce's most striking characteristics as follows:

"Professor Peirce was much more than a mathematician. Like many men of the time, he was a warm lover and a cordial hater. It could not always be guessed which side of a disputed question he would take; but one might be fairly sure that he would be at one extreme or the other. As a speaker and lecturer he was very pleasing, neither impressive nor eloquent, and yet interesting from his earnestness and vivacity. For this reason it is said that he was once chosen to enforce the views of the university professors at a town meeting, where some subject of interest to them was coming up for discussion. Several of the professors attended the meeting, and Peirce made his speech. Then a townsman rose and took the opposite side, expressing the hope that the meeting would not allow itself to be dictated to by these nabobs of Harvard College. When he sat down, Peirce remained in placid silence, making no reply. When the meeting broke up, some one asked Peirce why he had not replied to the man. 'Why! did you not hear what he called us? He said we were nabobs! I so enjoyed sitting up there and seeing all the crowd look up to me as a nabob that I could not say one word against the fellow.'"

An estimate by one who was not a scientist may be added. In a centennial address Wendell Phillips referred [5] to Peirce as "the largest natural genius, the man of the deepest reach and firmest grasp and widest sympathy, that God has

[1] E. W. Emerson, *The Early Years of the Saturday Club*, Boston and New York, 1918, pp. 100–101.

[2] *Nation*, New York, October 14, 1880.

[3] Judge Addison Brown wrote that "Professor Peirce seemed a poet in a mathematical dream, his mind so preoccupied, as it were gazing at the stars" (*Annals of the Harvard Class of 1852*, by G. W. Edes, Cambridge, 1922, p. 322; there are references to Peirce on several other pages).

[4] Boston, 1903, pp. 77–78; see also pp. 276–277.

[5] *The Scholar in a Republic. Address at the Centennial Anniversary of the Phi Beta Kappa Society of Harvard College*, Boston, 1881, p. 13.

given to Harvard in our day,—whose presence made you the loftiest peak and farthest outpost of more than mere scientific thought,—the magnet who, with his twin Agassiz, made Harvard for forty years the intellectual Mecca of forty States."

Andrew P. Peabody, '26, who was preacher to the University and professor of Christian morals at Harvard for the last two decades of Peirce's service, has devoted several pages of his *Harvard Reminiscences* [1] to Peirce. In referring to the last few years of Peirce's life he tells how he

> "had for his pupils only young men who were prepared for profounder study than ever entered into a required course, or a regularly planned curriculum; but he never before taught so efficiently, or with results so worthy of the mind and heart and soul, which he always put into his work. His students were inflamed by his fervor, and started by him on the eager pursuit of the eternal truth of God, of which mathematical signs and quantities are the symbols."

It was not alone "young men" whom Peirce was willing to direct, as the following extract from a letter [2] written in 1879 shows: "I will do the same for the young women that I do for the young men. I shall take pleasure in giving gratuitous instruction to any person whom I find competent to receive it. I give no elementary instruction, but only in the higher mathematics."

The pall-bearers at Peirce's funeral were President Eliot, Ex-President Thomas Hill, C. P. Patterson (Superintendent of the Coast Survey), J. J. Sylvester,[3] J. Ingersoll Bowditch, Simon Newcomb, Joseph Lovering, Andrew P. Peabody, and his classmates, James F. Clarke and Oliver Wendell Holmes.

At a meeting of the President and Fellows of Harvard College, October 11, 1880, the entry made upon the records regarding Peirce stated that

> "The University must long lament the loss of an intelligence so rare, an experience so rich, and a personal influence so strong, as his.
>
> "As a teacher, he inspired young minds with a love of truth, and touched them with his own enthusiasm; as a man of science, his attainments and achievements and his public services have reflected honor upon the University and the country."

What has been indicated above, coupled with the bibliography which follows, provides material necessary for forming an intelligent opinion as to the activities, personality, brilliant and powerful mind, and "wonderfully stimulating influence" of one of the most eminent and original scientists that America produced in the last century. In this country Peirce was the leading mathematician of his time, and a pioneer in achieving notable mathematical research, some of it anticipating work by well-known Europeans of later date. It is interesting to speculate as to the possible publication harvest if Peirce had been able throughout his career constantly to meet his mathematical equals or peers, and if he had

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[1] Boston, 1888, pp. 180–186.

[2] From a letter in the possession of D. E. Smith of Columbia University.

[3] "When Professor Sylvester was called [1876] to the chair of mathematics in the Johns Hopkins University, Professor Peirce of Harvard, being asked what he thought would be the opinion of American mathematicians respecting the new appointment, replied that no American mathematician had a right to have any opinion on the subject, except himself, and one of his old pupils, a distinguished professor of mathematics in one of our leading colleges." L. A. Wait, "Advanced Instruction in American Colleges," *The Harvard Register*, vol. 3, p. 127, 1880; see also page 119.

had a capable disciple always at hand to put his ideas on paper in a form suitable for publication.

In a passport dated May 14, 1860, Peirce is described as aged 51, of height 5 feet $7\frac{3}{4}$ inches, and with high forehead, hazel eyes, straight nose, regular mouth, round chin, brown hair, light complexion and oval face. A ticket dated Crystal Palace, London, October 5, 1873, gives his weight as 190 pounds.

Portraits of Peirce may be seen in the following sources:

Daguerreotype in possession of Peirce's grandson, Mr. Benjamin P. Ellis. This is the earliest known portrait of Peirce and is now reproduced for the first time. Probably it dates from about 1845.

Painting in the Faculty Room, University Hall, Harvard University, executed by J. A. Ames (1816–1872) about 1853(?). It was Mrs. Peirce's favorite portrait, and was presented to Harvard after her death in 1888. Reproduced herewith; also in *Harvard Alumni Bulletin*, January 3, 1924, vol. 26, p. 375.

Painting executed in 1857 by Daniel Huntington (1816–1906) purchased at auction by George A. Plimpton of New York City, in 1924, and presented to Harvard University. It is now reproduced for the first time.

Photograph by Whipple and Black, Boston, in 1858, in the class of 1829 album (in ms.), Harvard University Library.

Steel engraving, *The Mathematical Monthly*, ed. by J. D. Runkle, vol. 2, July 1860, facing page 329. Engraved by H. W. Smith from a daguerreotype by Southworth and Hawes in 1860, a "most accurate likeness." The same engraving appeared as frontispiece to *Annual of Scientific Discovery . . . for 1870*, Boston, 1870.

Very interesting reproduction of a photograph, full-length, of Peirce working at the blackboard and apparently taken about 1865; *Centennial Celebration of the United States Coast and Geodetic Survey*, Washington, 1916, p. 153. Copied in *The Scientific Monthly*, vol. 3, December, 1916, p. 618. Also on a plate opposite page 96 of *The Early Years of the Saturday Club 1855–1870*, by E. W. Emerson, Boston, 1918. Also now reproduced.

Photograph taken about 1872, nearly full-length, and reproduced in *The Harvard Book* by F. O. Vaille and H. A. Clarke, vol. 1, Cambridge, 1875, opposite page 172. Copied in *The Outlook*, New York, vol. 59, 1898, p. 323. Also in *Universities and Their Sons*, vol. 2, Boston, 1899, p. 228.

Reproduction of photograph, taken about 1872, F. C. Peirce, *Peirce Genealogy*, Worcester, 1880, oppo. p. 118.

Reproduction of photograph by G. W. Pach, New York, 1879, in *Harper's New Monthly Magazine*, March 1879, vol. 58, p. 508. Also in this MONTHLY, *1895*, oppo. p. 173.

Photograph, taken about 1879, reproduced as a steel engraving frontispiece in B. Peirce, *Ideality in the Physical Sciences*, 1881. Also in *Amer. Jl. Mathematics*, vol. 24, 1902, frontispiece. Woodcut of the same, apparently, in

The Harvard Register, May 1880, vol. 1, p. 91; in *Popular Science Mo.*, vol. 18, 1881, oppo. p. 578; and in M. King, *Benjamin Peirce . . . A Memorial Collection*, 1881.

Many other unpublished photographs of Benjamin Peirce are in Mr. Ellis's possession. There are also at least two other paintings of Peirce owned by grandchildren, one an inferior portrait by Wite and the other by a Miss Whitney from a photograph.

VI. The Writings of Peirce.

By R. C. Archibald.

[Solutions of problems.]
The Mathematical Diary, New York, vol. 1, 1825, ed. by R. Adrian, pp. 281, 286.

[Solutions of problems, and a problem for solution.]
The Mathematical Diary, New York, vol. 2, ed. by J. Ryan; no. X, 1828, p. 89; no. XI, 1830, pp. 116, 118; no. XIII, 1832, pp. 211–212, 216, 237, 244, 246, 310.

Laplace, *Mécanique Céleste translated with a commentary* by Nathaniel Bowditch.
Boston, 4 volumes, 1829–1839.
B. Peirce revised the entire work and detected many errata (compare the memoir in volume 4, pp. 61 and 140).

The American Almanac and Repository of Useful Knowledge. [For the years 1830–1851, the astronomical department was under the direction of B. Peirce.]
Boston, 1829–1850.

On perfect numbers.
The Mathematical Diary, vol. 2, no. XIII, 1832, pp. 267–277.
Euler showed that every even perfect number is expressible in the Euclidean form $2^{n-1} \cdot p$ where $p = 2^n - 1$ is a prime. In Peirce's paper it is shown that there can be no odd perfect number "included in the form a^r, $a^r b^s$, $a^r b^s c^t$, where a, b, and c are prime numbers and greater than unity." In his *History of the Theory of Numbers*, vols. 1–3, 1919–1923, L. E. Dickson does not mention this paper. He does record that in 1844 "V. A. Lebesgue stated that he had a proof that there is no odd perfect number with fewer than four distinct prime factors." We now see that an American mathematician published a proof of this theorem twelve years earlier.

An elementary Treatise on Plane Trigonometry with its applications to heights and distances, navigation and surveying.
Cambridge and Boston, Munroe, 1835, 7 + 90 pp. + 1 fold. pl.

First part of an elementary Treatise on Spherical Trigonometry.
Boston, J. Munroe & Co., 1836, 4 + 71 pp. + 1 fold. pl.

An elementary Treatise on Sound: being the second volume of a course of natural philosophy designed for the use of high schools and colleges. Compiled by Benjamin Peirce.
Boston, J. Munroe & Co., 1836, 56 + 220 pp.; diagrs. on 10 fold. pl.
"The Catalogue of works relating to sound," pp. v–lvi, is interesting, and the list of titles connected with musical matters, pp. xvii–xlviii, valuable. The work is based on J. F. W. Herschel's treatise on sound, in the *Encycl. Metropolitana*. Reviewed in *The New York Review*, vol. 4, 1839, pp. 164–176.

[Problems and solutions.]
The Mathematical Miscellany, ed. by C. Gill, New York, no. II: 1836, pp. 81–94, 94–97, 101–107; III: 1837, pp. 160–163 (194); IV: 1837, pp. 210–211, 233–234, 251–255, 258; V: 1838, pp. 289–290, 296, 309–311, 316–318, 327; VI: 1838, 359, 362–363, 383–387, 392–395, 397, 399; vol. 2, no. VII: 1839, pp. 16, 33–34, 42–47, 61, 63; VIII: 1839, 91, 92, 97–98, 110–113, 114, 117.
For the question discussed vol. 2, pp. 97–98, compare *Sphinx Œdipe*, vol. 8, 1913, pp. 93–94. It seems probable that many problems proposed by "P." in the *Mathematical Miscellany* were due to Peirce. See, for example, vol. 1, pp. 53, 55, 109, 110, 257–258, etc.

Benjamin Peirce, 1865 (?)

An elementary Treatise on Algebra: to which are added exponential equations and logarithms.
Boston, J. Munroe & Co., 1837, 10 + 276 pp.
Another edition, 1842, 4 + 284 pp.; 1843; fifth ed., 1845; sixth ed., 1846; other editions or reprints, 1850, 1851, 1855, 1858, 1860; and published by W. H. Dennett in 1864, 1865, 1870.

An elementary Treatise on Plane and Solid Geometry.
Boston, J. Munroe & Co., 1837, 20 + 159 pp. + 6 fold. pls.
Other editions or reprints, 20 + 3–150 p. + 6 fold. pls., 1841, 1847, 1851, 1853, 1855, 1857, 1860; and publ. by W. H. Dennett in 1863, 1865, 1866, 1867, 1868, 1869, 1870, 1871, 1872.

An account of Mr. Talbot's "Researches in the integral calculus" [*Philosophical Transactions*, vols. 54–55, 1836–37].
The Mathematical Miscellany, ed. by C. Gill, vol. 1, no. VI, 1838, pp. 404–411.

[Anonymous review of Laplace's *Mécanique Céleste*, vols. 1, 3–5; N. Bowditch's translation, vols. 1–4; and A. Young, D. A. White, and J. Pickering on Bowditch.]
N. Amer. Rev., vol. 44, 1839, pp. 143–180.

[Anonymous review of J. Pickering, D. A. White, and A. Young on N. Bowditch.]
The New York Review, vol. 4, 1839, pp. 308–323.

An elementary Treatise on Plane Geometry. . . . Printed for the use of the blind.
Boston, At the Press of the Perkins Institution and Massachusetts Asylum, 1840. 71 pages of definitions and demonstrations + 19 pages embossed diagrams.
In Boston line type; size 11 x 10 inches.

An elementary Treatise on Plane and Spherical Trigonometry, with their application to navigation, surveying, heights and distances and spherical astronomy, and particularly adapted to explaining the construction of Bowditch's Navigator and the Nautical Almanac.
Boston, J. Munroe & Co., 1840, 4 + 428 pp. + 5 fold. pls.
Third edition with additions, 1845, 4 + 449 pp. + 4 fold. pls.
New edition revised, with additions, 1852, 6 + 360 pp. + 5 fold. pls.
Another edition or reprint, 1861.
Pages 317–357 + plates 4–5 were reprinted in 1852 with the following title page: "*The chapter on Eclipses extracted from Peirce's Spherical Astronomy. For the Use of the Nautical Almanac.*

An elementary Treatise on Curves, Functions, and Forces. Volume first; containing analytical geometry and the differential calculus, 1841: *Volume second; containing calculus of imaginary quantities, residual calculus, and integral calculus*, 1846.
Boston, James Munroe & Co., 8 + 304 pp. + 14 pls. + 8 + 290 pp. + fold. pl.
New edition of volume 1, 1852. 7 + 301 pp. + 14 pls.
A third volume of this work, dealing with the applications of analytic mechanics, was projected, but not published. Probably the treatise issued in 1855 took its place. The following text by J. M. Peirce is based on this treatise: *A Text-book of Analytic Geometry; on the basis of Professor Peirce's Treatise.* Cambridge, 1857, 7 + 228 pp. + 6 fold. pl. Extracts from the preface: "I would acknowledge, in closing, my obligations for the aid and encouragement which I have received from others. Professor Peirce has given me the benefit of his advice in repeated instances. Whatever merit the book may have is owing, in a great degree, to the assistance of Mr. C. W. Eliot, who, besides many less definite, but important services, has read and criticised a considerable part of the manuscript before it was sent to the press."
In 1845 the following work was published in Boston by Thomas Hill, afterwards president of Harvard College: *An Elementary treatise on Arithmetic, designed as an Introduction to Peirce's Course of Pure Mathematics* [that is the series of works published 1835–1846] *and as a sequel to the Arithmetics used in the High Schools of New England.*

The Cambridge Miscellany of Mathematics, Physics, and Astronomy, no. I (April, 1842), edited by B. Peirce; no. II (July, 1842), no. III (October), no. IV (last, January, 1843), edited by B. Peirce and J. Lovering.
Boston, James Munroe & Co., 1–48 + 49–96 + 97–144 + 145–192 pp. + 3 plates.

Contributions by B. Peirce: Problems and Solutions, pp. 23–24, 58, 60–61, 66–72, 97, 102, 119, 145, 149, 155, 156, 159, 168; "American astronomical and magnetic observers," pp. 25–28; "Distances of the fixed stars," pp. 28–31; "Meteors," pp. 44–46; "Varieties of climate," pp. 46–47; "The barometer," p. 48; "On Espy's theory of storms," pp. 141–144. Joseph Lovering (1813–92) was teacher of mathematics and natural philosophy at Harvard for the fifty years prior to 1888.

[Anonymous review of S. C. Walker's work on Meteors.]
N. Amer. Rev., vol. 56, 1843, pp. 409–435.

On the perturbation of meteors approaching the earth.
Amer. Philos. Soc., Trans. n.s., vol. 8, 1843, pp. 83–86.
A letter dated Dec. 24, 1840, to S. C. Walker. Presented to the Society, January 15, 1841.

Bowditch's Useful Tables. [Preface by J. I. Bowditch, pages iii–v; "Remarks of Professor Pierce" (*sic*), pp. vii–viii.]
New York, E. & G. Blunt, 1844. Other editions were published in 1856, 1859, 1863, and 1866. An edition was issued by the U. S. Bureau of Navigation in 1885. Recent editions (such as the one for 1911) do not contain Peirce's remarks.
The tables in question are taken from Nathaniel Bowditch's *Practical Navigator*.

[Elements of the] third comet of 1845.
Amer. Jl. Sci., vol. 49, 1845, pp. 220–221.

The latitude of Cambridge Observatory, in Massachusetts, determined, from transits of stars over the prime vertical, observed during the months of December, 1844, and January, 1845, by William C. Bond, James D. Graham, and George P. Bond.
Amer. Acad., Memoirs, Boston, n.s., vol. 2, 1846, pp. 183–203.

[Orbits for Bond's comet.]
Boston Courier, March 27, 1846, p. 2, col. 2.
Also in *Amer. Jl. Sci.*, n.s., vol. 1, 1846, p. 348.
Letter dated March 26, 1846.

The Perturbations of Uranus.
Boston Courier, April 30, 1847, p. 2, col. 3.
Also in *Amer. Acad., Proc.*, vol. 1 (1846–48), 1848, pp. 144–145.
Also in *Amer. Jl. Sci.*, n.s., vol. 4, 1847, pp. 132–133.
Communication dated April 29, 1847.

Mass of Neptune.
Boston Courier, October 25, 1847, p. 2, cols. 2–3.
Also in *National Intelligencer*, Washington, October 26, 1847, p. 2, col. 5.
Leverrier's reply is in the *Intelligencer*, March 10, 1848, p. 2, cols. 5–6, and in the *Boston Daily Advertiser*, March 20, p. 1, cols. 5–6. To this Peirce replied in a letter dated March 13, 1848; this is in the *Intelligencer*, March 23, p. 1, col. 2; and in the *Advertiser*, March 24, p. 2, col. 2, "Leverrier and Mr. Peirce." The following letter from Asa Gray, the distinguished botanist then professor of natural history at Harvard, is dated March 26:

Dear Peirce

When I read, in the Daily, your letter, I was on the point of sitting down to write you a line to tell you that I think it a *perfect gem*, and the most beautiful contrast to the *Johnny Crapeau* vociferation of Le Verrier. I am perfectly charmed with its spirit, and all that I have heard speak of it have taken the same view. I am not alone, therefore, in the opinion that it does you the highest credit and it is just the style of reply calculated to place you at the greatest advantage. As one zealous for the highest interests and character of American Science and American *Savans*, I thank you most sincerely, and am

Faithfully yours,

A. Gray

Also in *The Siderial Messenger*, Cincinnati, vol. 2, pp. 28–29, 1847.
Letter dated October 22, 1847.

[Elements of an elliptic orbit of De Vico's fourth comet.]
Amer. Acad., Proc., vol. 1 (1846–48), 1848, pp. 39–42.

Notice of the computations of Mr. Sears C. Walker, who found that a star was missing in the *Histoire Céleste Française*, observed by Lalande on the 10th of May, 1795, near the path of the planet Neptune at that date, which may possibly have been this planet.
Amer. Acad., Proc., vol. 1 (1846–48), 1848, pp. 57–68 See also pp. 41–42.
Pages 57–65 appeared also in *The Siderial Messenger*, Cincinnati, vol. 1, pp. 125–128, 1847, under the title "The planet Neptune"; some parts of the other pages are given, in substance, on pages 85–86 of the same volume in an article entitled "Le Verrier's planet."
Pages 64–67 appeared as a quotation in *Amer. Jl. Sci.*, n.s., vol. 3, 1847, pp. 441–443.

[Review of Nichol's *Contemplations on the Solar System.*]
N. Amer. Rev., vol. 66, 1848, pp. 253–255.

[Formulæ for the perturbations of Neptune's longitude and radius vector.]
Amer. Acad., Proc., vol. 1 (1846–48), 1848, pp. 285–295.

[Investigations into the action of Neptune upon Uranus.]
Amer. Acad., Proc.,[1] vol. 1 (1846–48), 1848, pp. 332–342.

(*a*) Perturbations [of Neptune]; (*b*) Elements of Neptune; (*c*) Satellite of Neptune; (*d*) [Perturbations of Uranus].
Mo. Notices R. Astr. Soc., vol. 8 (1847–48), 1848, pp. 38–40, 128, 202–203.

Ueber die Störungen des Neptuns.
Astron. Nachrichten, vol. 27, cols. 215–218, 1848.

[Calculations on the perturbations of Uranus.]
Amer. Jl. Sci., n.s., vol. 5, 1848, pp. 435–436.

Development of the perturbative function of planetary motion.
Astr. Jl., vol. 1, pp. 1–8, 1849; pp. 31–32, 33–36, 1850.

Certain methods of determining the number of real roots of equations applicable to transcendental as well as to algebraic equations [abstract].
Amer. Assoc. Adv. Sci., Proc., vol. 1 (1848), 1849, pp. 38–39.

A. Guyot, *The Earth and Man.* Translated from the French by C. C. Felton. Boston, 1849.
Quotation from the preface: "Besides Prof. Felton, who read all the proof sheets, the author returns his acknowledgments to Professors Agassiz, Peirce and Gray who have had the goodness to revise portions of them."

On Fresnel's dioptric apparatus for lighthouses, by B. Peirce and J. Lovering.
Franklin Inst., Jl., s. 3, vol. 18, 1849, pp. 249–252.

"Poor Richard." Poor Richard's Almanac for 1850(–52) as written by Benjamin Franklin for the years 1733, 1734, 1735, (1736–1741). The astronomical calculations by . . . Benj. Peirce. New York, John Doggett, Jr., 1849(–51).

(*a*) On the connection of comets with the solar system; (*b*) On the relation between the elastic curve and the motion of the pendulum; (*c*) Mathematical investigation of the fractions which occur in phyllotaxis.
Amer. Assoc. Adv. Sci., Proc., vol. 2 (1849), 1850, pp. 118–122; 128–130; 444–447.
As to the topic discussed in (*c*) compare my articles on Golden section and the Fibonacci series in this MONTHLY, *1918*, 232–238; or better in Jay Hambidge, *Dynamic Symmetry*, New Haven, 1920, pp. 152–157. See also C. Wright, "On the phyllotaxis," *Astron. Jl.*, vol. 5, pp. 22–24, 1856.

(*a*) Note of Professor Peirce to the editor [demonstrating the parallelogram of forces]; (*b*) On the orbit of α Virginis regarded as a double star.
Astr. Jl., vol. 1, pp. 23, 138–139, 1850.

On the constitution of Saturn's rings.
Amer. Jl. Sci., vol. 12, 1851, pp. 106–108.
Also in *Astr. Jl.*, vol. 2, pp. 17–19, 1851.
German translation: Ueber die Beschaffenheit des Saturnringes, *Annalen der Physik und Chemie*, vol. 160, 1851, pp. 313–319.

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[1] There are numerous references in the *Proceedings* to papers read by Peirce but not published. See vol. 1 (1846–48), 1848, p. 185; vol. 2 (1848–52), 1852, pp. 111–2, 147, 235, 240, 250, 256, 258, 282, 289–90, 298, 310; vol. 3 (1852–57), 1857, pp. 8, 9, 28, 31, 67, 83.

Report of the Committee upon Prof. Mitchell's system of astronomical observations by Benjamin Peirce, chairman.
Amer. Assoc. Adv. Sci., Proc., vol. 5, 1851, pp. 69–71.

(*a*) [Report on the results of the U. S. Coast Survey, by B. Peirce for the Committee: B. Peirce, D. Treadwell, J. I. Bowditch, and J. Lovering]; (*b*) [Report by B. Peirce, E. N. Horford, and J. Lovering on a paper, entitled, "Description of the causes of the explosion of steam boilers, and of some newly-discovered properties of heat and other matters; for the purpose of showing that the application of steam for the production of motive force is susceptible both of immense improvement and economy," by James Frost]; (*c*) [On a new method of computing the constants of the perturbation function of planetary motion]; (*d*) Report by B. Peirce (in behalf of the committee D. Treadwell, B. Peirce, J. Lovering, H. L. Eustis, and M. Wyman) concerning C. H. Davis's paper on the deterioration of Boston harbor.
Amer. Acad., Proc., vol. 2 (1848–52), 1852, pp. 124–128; 129–130; 197–198; 288–289.

An account of Longstreth's lunar formula.
Amer. Assoc. Adv. Sci., Proc., vol. 6 (1851), 1852, pp. 143–144.

The American Ephemeris and Nautical Almanac for the year 1855 [vol. 1] (–1861).
Washington, 1852(–1858).
From the preface: "The theoretical department of the work has been placed under the special direction of Professor Benjamin Peirce, LL.D., and most of the calculations have passed under his final revision."

(*a*) Note upon the conical pendulum; (*b*) Criterion for the rejection of doubtful observations.
Astr. Jl., vol. 2, 1852, pp. 137–149; 161–163.
An elucidation of the second of these papers is given in the paper on the Criterion, published by Peirce in 1878. See further the notes on this subject in Section V, p. 13.

The semidiameters of Venus and Mars investigated. From the observations made with the mural-circle of the Naval Observatory at Washington during the years 1845 and 1846.
Astr. Jl., vol. 3, pp. 9–10, 1852.

Tables of the Moon; constructed from Plana's theory, with Airy's and Longstreth's corrections, Hansen's two inequalities of long period arising from the action of Venus, and Hansen's values of the secular variations of the mean motion and of the motion of the perigee. Arranged in form designed by . . . Benjamin Peirce.
Washington, For the use of the Nautical Almanac, 1853. 4to, 326 pp.
See also *Tables of the Moon's Parallax*, 1856.
The second edition (1865, 348 pp.) was "the same as the first, with the exception of the correction of typographical errors; the substitution of the Tables of the Moon's Parallax constructed from Walker's and Adams' formulas, in the place of the original Parallax Tables; and the addition of a Table adapted to a convenient modification of the method of computing the latitude, by Professor J. D. Runkle.
"A third edition will shortly be issued, of which the basis will still be Plana's theory, while the Tables will be corrected to conform to the new Solar Parallax, and the corrected elements of the Moon's orbit." (Preface; the third edition does not seem to have been published.)
The work was reviewed in *Mo. Notices R. Astr. Soc.*, vol. 14, pp. 26–32, 184. On page 32 it is remarked that "the arrangement . . . is the result of a plan devised by Professor Peirce. It is very clear and masterly, and is in every respect worthy of that eminent mathematician."

On longitudes from moon culminations.
Coast Survey, Report for 1853, Washington, 1853, app. 31, p. 84.

Address of Professor Benjamin Peirce, president of the American Association for the year 1853, on retiring from the duties of president. [Printed by order of the Association.]
[Cambridge], 1854, 17 p.
Also in *Amer. Assoc. Adv. Sci., Proc.*, vol. 8 (1854), 1855, pp. 1–17.
Also bound in limp cloth, gilt edges, with cover title *The Song of Geometry*, and with special title-page and dedication. The copy of the title page is as follows: *Ben Yamen's Song of Geometry, sung by the Florentine Academy, at the accession of Her Majesty the Queen, degraded into prose by Benjamin the Florentine*, Cloverden, 1854.
Also most of pages 2–5 appears on pages 105–108 of *The Early Years of the Saturday Club, 1855–1870*, by E. W. Emerson, Boston, 1918.

Residual differences between the theoretical and observed longitudes of Uranus, from the theories of Peirce, LeVerrier and Adams.
Amer. Phil. Soc., Proc., vol. 5, 1854, p. 16.

(*a*) Elements of the comet, 1854, III; (*b*) Quantities to be added to the solar ephemeris of the American Nautical Almanac to obtain that given by Hansen's solar tables with the obliquity of the ecliptic of the Nautical Almanac; (*c*) The investigation of the catenary upon a cone of revolution with a vertical axis.
Astr. Jl., vol. 4, pp. 7, 9, 27–29, 1854.

Report upon the determination of longitude by moon-culminations.
Coast Survey, Report for 1854, Washington, 1854, app. 36, pp. 108–120.

On the Adams prize-problem for 1856.
Astr. Jl., vol. 4, pp. 27–29, 1854.
At the conclusion of the article are the words "to be continued"; no continuation has been found.

[Report of the Committee of the American Academy of Arts and Sciences on a Program for organization of the Smithsonian Institution, December 8, 1847.]
Smithsonian Institution, Eighth Annual Report, Washington, 1854, pp. 148–155.
Also in *The Smithsonian Institution. Documents Relative to its Origin and History* (*Smithsonian Misc. Colls.*, vol. 17, Washington, 1879, pp. 964–970).
The Committee consisted of E. Everett (Chairman), Jared Sparks, Benjamin Peirce, H. W. Longfellow, and Asa Gray.

Physical and Celestial Mechanics . . . developed in four Systems of Analytic Mechanics, Celestial Mechanics, Potential Physics, and Analytic Morphology. Then the second title page: *A System of Analytic Mechanics.*
Boston, Little, Brown & Co., 1855, 39 + 496 pp. + a fold. pl.
This work on mechanics was intended as the first of a series of four volumes, the other three to be respectively on Celestial Mechanics, Potential Physics, and Analytic Morphology.
Extract from preface: "I have . . . reexamined the memoirs of the great geometers, and have striven to consolidate their latest researches and their most exalted forms of thought into a consistent and uniform treatise. If I have, hereby, succeeded in opening to the students of my country a readier access to these choice jewels of intellect, if their brilliancy is not impaired in this attempt to reset them, if in their new constellation they illustrate each other and concentrate a stronger light upon the names of their discoverers, and still more, if any gem which I may have presumed to add is not wholly lustreless in the collection, I shall feel that my work has not been in vain. The treatise is not, however, designed to be a mere compilation. The attempt has been made to carry back the fundamental principles of the science to a more profound and central origin; and thence to shorten the path to the most fruitful forms of research. See further comments in Section V of this monograph.
Reviewed in: *Christian Examiner*, Boston, vol. 64, 1858, pp. 276–293 [by Thos. Hill]; *N. Amer. Rev.*, vol. 87, 1858, pp. 1–21 [by T. Hill].
Another issue with new title-page, New York, Van Nostrand, 1865; also 1872.

On the method of determining longitudes by occultations of the Pleiades.
Coast Survey, Report for 1855, Washington, 1855, app. 42, pp. 267–274.

Six articles upon the Smithsonian Institution . . . together with the letters of Professor Peirce and Agassiz.
Boston Post, January 27, February 5, 7, 13, 21, 22, 1855.
Also as a pamphlet, Boston, Printed at the office of the *Boston Post*, 1855, 44 pp.
The letter of Professor Peirce was dated January 29, 1855. The articles were signed "N. P. D." The last three of the five paragraphs of Peirce's letter were published in *The Congressional Globe, Appendix* (33d Congress, 1853–55, House of Representatives, February 27, 1855), vol. 31, p. 285, and reprinted in *The Smithsonian Institution, Documents relative to its Origin and History* (*Smithsonian Miscellaneous Collections*, vol. 17), Washington, 1879, pp. 588–9, 619.

Letter of Professor Peirce to President Quincy, with two letters from Admiral Beaufort annexed to it, and a list of zenith stars from Professor Airy.
Annals of the Harvard Observatory, vol. 1, 1856, pp. xciv–xcv.
The letter was dated May 10, 1845.

Opening address of Professor Benjamin Peirce . . . President of the Association.
Amer. Assoc. Adv. Sci., Proc., vol. 7 (1853), 1856, pp. xvii–xx.

(*a*) Abstract of a paper on researches in analytic morphology. Transformation of curves; (*b*) Abstract of a paper upon the solution of the Adams prize problem for 1867; (*c*) Abstract of a paper on partial multipliers of differential equations; (*d*) Abstract of a paper upon the catenary on the vertical right cone; (*e*) Abstract of a paper upon the motion of a heavy body on the circumference of a circle which rotates uniformly about a vertical axis; (*f*) Abstract of a paper on the resistance to the motion of the pendulum; (*g*) Method of determining longitudes by occultations of the Pleiades.
Amer. Assoc. Adv. Sci., Proc., vol. 9 (1855), 1856, pp. 67–74, 97–102.

Tables of the Moon's Parallax, constructed from Walker's and Adams's formulæ, arranged as a supplement to the first edition of Peirce's Tables of the moon.
Washington, For the U. S. Nautical Almanac Office, 1856, pp. 303–329.

Working plan for the Foundation of a University.
Cambridge, Mass., 1856, 4 pp.
"Printed for private and confidential circulation among the advocates and patrons of the University" [Harvard].

On the determination of longitude by occultations of the Pleiades.
Coast Survey, Rept. for 1856, Washington, 1856, app. 24, pp. 191–197.

An investigation of the cases of complete solution by integration by quadratures of the problem of the motion of a material point acted upon by forces which emanate from a fixed axis.
Astr. Jl., vol. 5, pp. 38–39, 1857.

[Report, dated June 29, 1857, by a committee consisting of Benjamin Peirce (chairman), Louis Agassiz, B. A. Gould, and E. N. Horsford, on spiritualistic phenomena presented by a Dr. Gardner in an attempt to win a $500 prize offered in 1857 by the *Boston Courier*.]
Boston Courier, July 1, 1857, p. 2, col. 2.
Also in Epes Sargent, *Planchette; or the Despair of Science*, Boston, 1869, pp. 10–11; see also p. 13. See further, "The Cambridge professors" in T. L. Nichols, *A Biography of the Brothers Davenport*, London, 1864, pp. 83–91; and G. A. Redman, *Mystic Hours; or Spiritual Experiences*, New York, 1859, pp. 307–317.

Determination of longitudes by occultations of the Pleiades and solar eclipses.
Coast Survey, Rept. for 1857, Washington, 1857, app. 29, pp. 311–314.

(*a*) Note on the Red Hill catalogue of circumpolar stars; (*b*) Note on the extension of Lagrange's theorem for the development of functions.
Astr. Jl., vol. 5, pp. 137, 164, 1858.

On the formation of continents.
Canadian Jl. of Industry, Science, and Art, Canadian Institute, Toronto, n.s., vol. 3, 1858, pp. 69–70.

(*a*) Problem; (*b*) Propositions on the distribution of points on a line; (*c*) Note on two symbols.
Mathematical Monthly, ed. by J. D. Runkle, vol. 1, pp. 11, 16–18, 58, 60, 1858; 167–168, 170, 1859.

Cotidal lines of an inclosed sea, derived from the equilibrium theory.
Coast Survey, Rept. for 1858, Washington, 1858, app. 30, pp. 210–213.

Defence of Dr. Gould by the Scientific Council of the Dudley Observatory [*Albany, N. Y.*].
Albany, Weed, Parsons & Co., 1858, 93 pp.
Signed by Joseph Henry, A. D. Bache, Benjamin Peirce, Dudley Observatory, July 1858. See also *The Dudley Observatory and the Scientific Council, Statement of the Trustees*, Albany, 1858; letters of Peirce, pp. 54, 101, 102. Also, *A Key to the "Trustee's Statement." Letters to the Majority of the Trustees of the Dudley Observatory, showing the Misrepresentation, Garblings, Perversions of their Misstatements*, by George H. Thacher. Albany, Atlas & Argus, Oct. 1858, p. 126.
Third edition, Albany, 1858.

On the theory of the comet's tail.
Astr. Jl., vol. 5, pp. 186–188, 1858; vol. 6, pp. 50–56, 1859.

[Resolutions by B. Peirce, at meeting of A. A. A. S., in Springfield, Mass., voting thanks to ladies of Springfield.]
The Atlas and Daily Bee, Boston, vol. 34, Aug. 11, 1859, p. 1, col. 7.

[W. C. Bond, director of Harvard Observatory; obituary notice.]
Amer. Acad., Proc., vol. 4 (1857–60), 1860, pp. 163–166.

Lettre addressée à M. le président de l'Académie des Sciences sur la constitution physique des comètes.
Comptes Rendus de l'Académ. d. Sc., vol. 51, 1860, pp. 174–176.

(*a*) [Abstract of a memoir on the peculiarities of astronomical observers;] (*b*) Memoir upon the tail of Donati's Comet.
Amer. Acad., Proc., vol. 4 (1857–60), 1860, pp. 197–199, 202–206.

Cyclic Solutions of the school-girl puzzle.
Astr. Jl., vol. 6, pp. 169–174, 1860.

The problem here discussed is the following: "A given number, f, of girls are required to walk in a given number, g, of ranks, of which each rank consists of a given number, k, of girls; subject to the condition that each girl is to walk once, and only once, in the same rank with every other girl." This important paper is apparently one of a series inspired by T. P. Kirkman's problem published in the *Lady's and Gentleman's Diary* for 1850, p. 48: "Fifteen young ladies in a school walk out thrice abreast for seven days in succession; it is required to arrange them daily, so that no two shall walk twice abreast." An account of this problem and some references to the literature are given in W. W. R. Ball, *Mathematical Recreations*, tenth edition, London, 1922, pp. 193–223; see also *Messenger of Mathematics*, vol. 41, 1911, pp. 33–56; *Jahrbuch über die Fortschritte der Mathematik*, 1911, p. 250, and W. Ahrens, *Mathematische Unterhaltungen und Spiele*, vol. 2, Leipzig, 1919, pp. 102–117.

(*a*) Report upon the determination of the longitude of America and Europe from the solar eclipse of July 28, 1851; (*b*) Report on an example for the determination of longitudes by occultations of the Pleiades.
Coast Survey, Rept. for 1861, Washington, 1861, (*a*) app. 16, pp. 182–195; (*b*) app. 17, pp. 196–221.

(*a*) Abstract of a memoir upon the attraction of Saturn's ring; (*b*) Upon the system of Saturn.
Amer. Acad., Proc., vol. 5 (1860–62), 1862, pp. (*a*) 353–354; (*b*) 379–380.

(*a*) On the computations of the occultations of the Pleiades for longitude; (*b*) Upon the tables of the moon used in the reduction of the Pleiades.
Coast Survey, Rept. for 1862, Washington, 1862; (*a*) app. 12, pp. 155, 156; (*b*) app. 13, pp. 157, 158.

Report upon the occultations of the Pleiades in 1841–42.
Coast Survey, Rept. for 1863, Washington, 1863, app. 17, pp. 146–154.

On the computations for longitudes by occultations of the Pleiades.
Coast Survey, Rept. for 1864, Washington, 1864, app. 11, p. 114.

(*a*) Report on the progress of determining longitude from occultations of the Pleiades [continued, compare *Report for 1863*]; (*b*) Method of determining the corrections of lunar semi-diameter, mean place, ellipticity of orbit, longitude of perihelion, coëfficient of annual parallax, and longitude of Europe and America from the occultation of the Pleiades.
Coast Survey, Rept. for 1865, Washington, 1865; (*a*) app. 12, pp. 138–146; (*b*) app. 13, pp. 146–149.

On the lunar bolis.
Amer. Acad., Proc., vol. 6 (1862–65), 1866, p. 36.

The Saturnian system.
Nat. Acad. Sci., Mem., vol. 1, 1866, pp. 263–286.

Coast Survey, Report, B. Peirce, Superintendent [For the years 1867–1873].
House Executive Documents, Washington, 1867–1873.
Details concerning the exact number of these reports in the *Documents* and the number of pages in each report may be found in B. P. Poore, *A Descriptive Catalogue of the Government Publications of the United States, 1774–1881,* Washington, 1885. See also E. L. Burchard, *List and Catalogue of The Publications issued by the U. S. Coast and Geodetic Survey, 1816–1902,* Washington, 1908.

Obituary on Alexander Bache.
Coast Survey, Rept. for 1867, Washington, 1867, app. 19, p. 330.

Communication of vibration.
Amer. Assoc. Adv. Sci., Proc., vol. 16 (1867), 1868, pp. 17–18.

Report on Weights and Measures.
Washington, Coast Survey, 1869, 4 pp.
Report upon the progress made in the construction of metric standards of length, weight, and capacity, in pursuance of a joint resolution of Congress of July 27, 1866.

The solar eclipse of December 22, 1870.
Coast Survey, Rept. for 1870, Washington, 1870, app. 16, pp. 229–232.

Linear Associative Algebra (Lithographed).
Washington City, 1870, 153 pp.
Edition limited to 100 copies issued through "labors of love" by persons engaged on the Coast Survey. This work was developed from papers read before the National Academy of Sciences, 1866–1870.
New edition, with addenda and notes by C. S. Peirce, son of the author.
Amer. Jl. Math., vol. 4, 1881, pp. 97–229.
Reprinted, New York, Van Nostrand, 1882, 4 + 133 pp.
This contains pp. 120–125, a reprint of: (1) his article on the uses and transformations of linear algebra, published in 1875; and (2) C. S. Peirce's notes, pp. 125–133, which appeared at the same time.
On page 656, volume 3 (1869) of R. P. Graves's *Life of Sir William Rowan Hamilton,* occurs the following with reference to Peirce's work as it appeared in the *American Journal:* "The author of this Paper in a *note,* on p. 105, makes objection to Quaternions on the ground of the treatment of imaginaries. A reply to this objection may, I believe, be gathered from what will be found stated by Sir William Hamilton in pages 578, 579 of vol. II and pages 84, 85 of vol. III of this work, as well as *passim* in the correspondence with Professor De Morgan."
See notes in Section V, pp. 15–16.

[Problem proposed.] Given the skill of two billiard players at the three-ball game, to find the chance of the better player gaining the victory if he gives the other a *grand discount.*
Our Schoolday Visitor, Philadelphia, vol. 15, 1871, p. 220, problem 108.
Also as problem 71 in *The Mathematical Visitor,* vol. 1, p. 46, 1878; solution by the proposer on p. 69, 1879. Compare Peirce's paper, "Probabilities at the three-ball game of billiards," 1877.

Observations of the eclipse of December 22, 1870, at Catania.
Boston Daily Advertiser, , 1871.
Also in *Amer. Jl. Sci.,* s. 3, vol. 1, 1871, p. 155*.
Letter dated Catania, Dec. 22, 1870.

On the mean motions of the four outer planets.
Amer. Jl. Sci., s. 3, vol. 3, 1872, pp. 67–68.
From a letter to H. A. Newton, dated December 13, 1871.

Harbor of New York: its Condition, May, 1873. Letter . . . to the Chamber of Commerce of New York, with the report of Prof. Henry Mitchell on the Physical Survey of the Harbor.
New York, Press of Chamber of Commerce, 1873. 3–38 pp. + 8 charts and tables.
The letter occupies pages 3–5.

[On the formation of the shell of the earth by shrinkage.]
Amer. Acad., Proc., vol. 8 (1868–73), 1873, pp. 106–108.

Benjamin Peirce, 1879

The rotation of the planets as a result of the nebular theory.
Nature, vol. 8, 1873, pp. 392–393.
Reprint of a report of the 1873 meeting of the A. A. A. S., in the *New York Tribune*.

Ocean lanes for steamships.
Amer. Acad., Proc., vol. 9 (1873–74), 1874, pp. 228–230.

On the uses and transformations of linear algebras.
Amer. Acad., Proc., vol. 10, 1875, pp. 395–400.
Also in *Amer. Jl. Math.*, vol. 4, 1881, pp. 216–221; in this form reprinted in *Linear Associative Algebra*, 1882, pp. 120–125.

A new system of binary arithmetic.
Coast Survey, Rept. for 1876, Washington, 1876, app. 6, pp. 81–82.

The conflict between science and religion.
Unitarian Review, Boston, vol. 7, 1877, pp. 656–666.
Also reprinted with cover title, Boston, 1877, 12 pp.
A discourse delivered in the First Church, Boston, May 6, 1877.

Qualitative algebra.
Johnson's New Universal Cyclopædia, New York, vol. 3, 1877, pp. 1487–88.

(*a*) Probabilities at the three-ball game of billiards; (*b*) on Peirce's criterion.
Amer. Acad., Proc., vol. 13 (1877–1878), 1878, pp. (*a*) 141–144; (*b*) 348–351.
The second of these papers was in elucidation of the paper on the same subject published in 1852. Compare page 13 of this monograph.

The National Importance of Social Science in the United States. An address delivered by Professor Benjamin Peirce, at the opening of the session of the American Social Science Association at Cincinnati, 18 May, 1878.
Boston, Little, Brown & Co., 1878, cover-title, 16 p.
Also in *Journal of Social Science*, no. 12, 1880, pp. xii–xxi.

[Problem proposed] 5564. Find the probabilities at a game of a given number of points, which is played in such a way that there is only one person who is the actual player, and when the player is successful he counts a point, but when he is unsuccessful, he loses all the points he has made and adds one to his opponent's score.
Educational Times, London, vol. 31, 1878, p. 88; solution, pp. 135–136.
Also in *Mathematical Questions with Solutions from the Educational Times*, vol. 29, 1878, pp. 72–73.
Also as problem 66 in *The Mathematical Visitor*, Erie, Pa., vol. 1, p. 45, 1878; Peirce's solution is given on page 66, 1879.
Also in E. J. Boudin, *Leçons de Calcul des Probabilités*, edited by P. Mansion, Paris, 1916, pp. VIII and 36 ff. Several solutions of the problem are given, one by A. Claeys, another by A. Demoulin, and its connections with important theory are set forth.

[Problem proposed] 5968. If two bodies revolve about a centre, acted upon by a force proportional to the distance from the centre, and independent of the mass of the attracted body, prove that each will appear to the other to move in a plane, whatever be the mutual attraction.
Educational Times, vol. 32, 1879, p. 152; solutions, vol. 33, 1880, pp. 141 (by C. J. Munro) and 309 (by Asaph Hall).
Also in *Mathematical Questions* . . . , vol. 33, 1880, p. 91; vol. 34, 1881, p. 111.
Also as problem 145 in *The Mathematical Visitor*, vol. 1, p. 84, 1879; quaternion solution by the proposer, and a solution by De V. Wood, p. 146, 1880.

Internal constitution of the earth.
Coast Survey, Rept. for 1879, Washington, 1879, app. 14, p. 201.

[Problems proposed] (*a*) 174. To find by quadratic equations a triangle of which the angles are given and the distances of the vertices from a given point in the plane of the triangle. (*b*) 202. Find a curve which is similar to its own evolute.
The Mathematical Visitor, vol. 1, pp. 99, 116, 1880; solutions of 174 by W. Hoover and W. Siverly, p. 174, 1881; solution of 202 by A. S. Christie, vol. 2, pp. 21–2, 1882.

Propositions in cosmical physics.
Amer. Acad., Proc., vol. 15, 1880, p. 201.

The intellectual organization of Harvard University.
The Harvard Register, April 1880, vol. 1, p. 77.

Ideality of the Physical Sciences. Edited by J. M. Peirce.
Boston, Little, Brown & Co., 1881, 7 + 9–211 pp. Portrait frontispiece (fine steel engraving).
The editing consisted in verbal changes and the addition of footnotes and the appendix (pp. 195–211).
Reprint, 1883.
This volume contains the six lectures delivered by B. Peirce in February and March, 1879, in a Lowell Institute course, Boston, Mass. They were also given, January 20 to February 5, 1880, at the Peabody Institute, Baltimore. The lectures are entitled: 1. Ideality in Science; 2. Cosmogony; 3. From nebula to star; 4. Planet, comet, and meteor; 5. The cooling of the earth and the sun; 6. Potentiality. The dedication is as follows:

I dedicate
these lectures
To my wife
with my whole heart.
Benjamin Peirce.

Cambridge, 790320

The last of these lectures was quoted under the heading, "Prof. Peirce on the spiritual body" in *Banner of Light*, Boston, May 3, 1879.

ADDENDA

¶ To the list of references given in footnote 1, page 9, and footnote 1, page 11, references may be given to J. E. Hilgard's obituary notice of Peirce in *Report of the Superintendent of the U. S. Coast and Geodetic Survey for the year ending June, 1881*, Washington, 1883, pp. 8–9; and to E. S. Holden, *Memorials of William Cranch Bond, director of the Harvard College Observatory 1840–1859, and of his son George Phillips Bond, director of the Harvard College Observatory 1859–1865*, San Francisco and New York, 1897. In this latter work are numerous references to Peirce in the index, some of them to material of special interest in portraying his character (pp. 26 and 163), and in exhibiting Leverrier's reaction (pp. 91–92) to Peirce's criticisms of his discovery of Neptune (cf. pp. 14 and 22 of this Monograph); Gauss's comments made in this connection in 1851 are recorded on page 109 where reference is also made to Gauss's two sons and a grandson in St. Louis. Peirce's notice of W. C. Bond (compare p. 27 of this Monograph) is reprinted on pages 43–46 of this work.

¶ In addition to the references to "Peirce's criterion" given on pages 13, 24, and 29, two more may be added, namely to: Mansfield Merriman's "List of writings, related to the method of least squares, with historical and critical notes," *Transactions of the Connecticut Academy*, vol. 4, 1877, especially pages 192–222; and to J. W. L. Glaisher's "On the law of facility of errors of observations and on the method of least squares," *Memoirs of the Royal Astronomical Society*, vol. 39, 1872, especially pages 120–121; compare with this page 397 of Glaisher's paper "On the rejection of discordant observations," *Mo. Notices R. Astr. Soc.*, vol. 33. The present Monograph seems to be the only one which refers to Peirce's 1878 paper on his criterion (which is followed by C. A. Schott's statement as to its value).

Gould's paper of 1855 was first published as follows: "Report to Professor A. D. Bache" . . . "containing directions and tables for the use of Peirce's criterion for the rejection of doubtful observations," *Coast Survey, Report for 1854*, Washington, 1855, pp. 131*–138*.

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